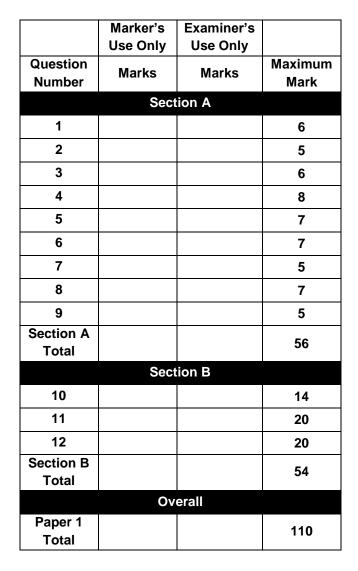
# Analysis and Approaches Higher Level for IBDP Mathematics Practice Paper Set 1 – Paper 1 (120 Minutes)

## **Question – Answer Book**

#### Instructions

- 1. This paper consists of **TWO** sections: A and B.
- 2. Attempt ALL questions. Write your answers in the spaces provided in this Question Answer Book.
- **3.** No calculator is allowed.
- 4. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.
- 5. Supplementary answer sheets and graph papers will be supplied on request.
- 6. Unless otherwise specified, ALL working must be clearly shown.
- Unless otherwise specified, numerical answers should be either EXACT or correct to 3 SIGNIFICANT FIGURES.
- 8. The diagrams in this paper are **NOT** necessarily drawn to scale.
- **9.** Information to be read before you start the exam:





## Section A (56 marks)

- **1.** There are 15 items in a data set. The sum of the items is 300.
  - (a) Find the mean.

[2] The variance of this data set is 9. Each value in the set is multiplied by -2.

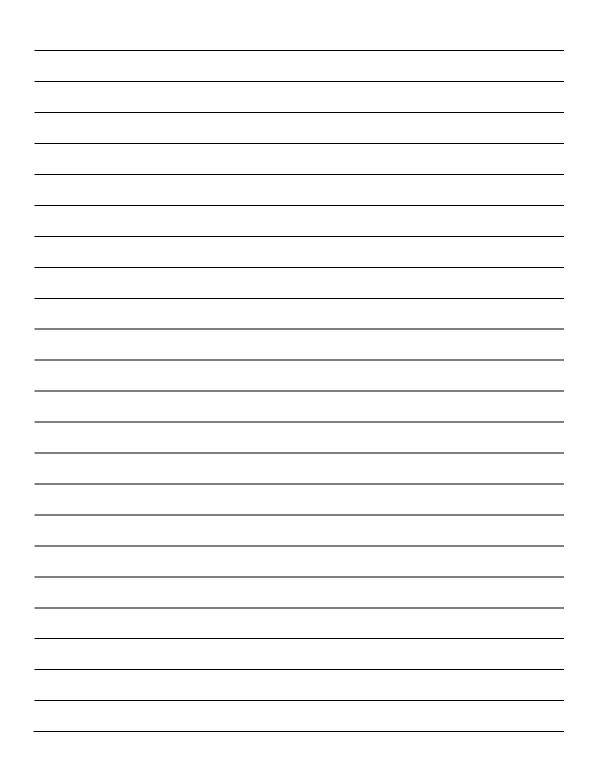
- (b) (i) Write down the value of the new mean.
  - (ii) Find the value of the new variance.
  - (iii) Hence, write down the value of the new standard deviation.




- **2.** A straight line  $L_1$  passes through the points (8, 0) and (24, 32).
  - (a) Find the equation of  $L_1$ , giving the answer in general form.
  - (b) The equation of another straight line,  $L_2$ , is given as x ay + 2021 = 0,  $a \in \mathbb{R}$ . If  $L_1$  and  $L_2$  are perpendicular, find a.

[2]

[3]



3.	(a)	Show that $(2n+1)^2 + (2n+3)^2 + (2n+5)^2 = 3(4n^2 + 12n + 11) + 2$ , where $n \in \mathbb{Z}$ .	[3]
	(b)	Hence, or otherwise, prove that the sum of the squares of any three consecutive odd numbers is greater than a multiple of $3$ by $2$ .	[3]
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© SE Production Limited All Rights Reserved 2021 4. Let  $f(x) = px^3 + qx^2 - 2x + 1$ . At x = 1, the slope of the normal of the curve of f is  $-\frac{1}{15}$ . It is given that  $f^{-1}(41) = 2$ , find the value of p and of q.

[8]


5. The graph of f is given by  $f(t) = a \sin b(t+2.5) + d$ , a > 0,  $t \ge 0$ .

When t = 2, there is a maximum value of 37, at P. When t = 11, there is a minimum value of -5. The graph of f is strictly decreasing at 2 < t < 11.

(a) Show that 
$$f(t) = 21\sin\frac{\pi}{9}(t+2.5)+16$$
.

[5] The graph of *f* is then transformed to the graph of *g* by a horizontal stretch of scale factor 3, followed by a translation of  $\begin{pmatrix} 17\\8 \end{pmatrix}$ . Let P' be the image of P.

(b) Find the coordinates of P'.

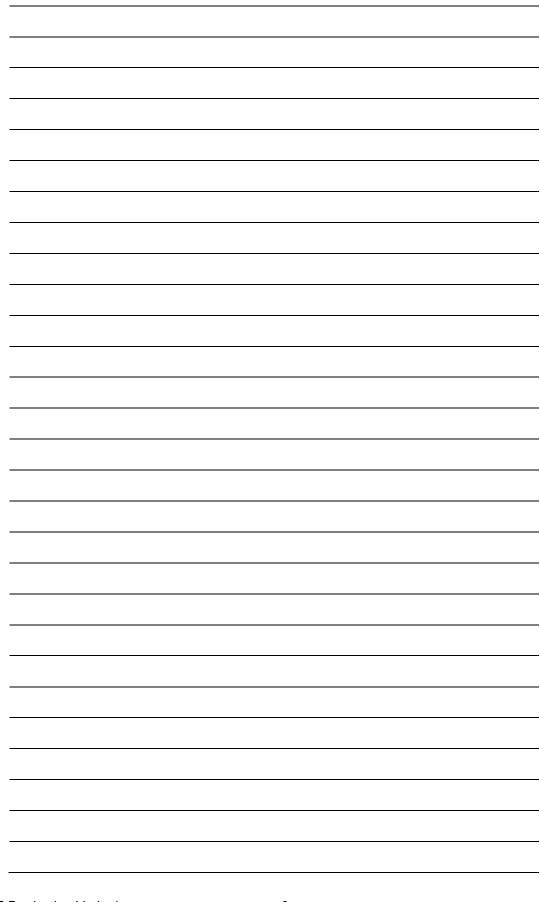
[2]



- 6. Consider the function  $f(x) = 4x^4 + 3x^2 1$ ,  $x \in \mathbb{R}$ . The graph of f is translated one unit to the right and then stretched vertically with scale factor 3 to form the function g(x).
  - (a) Express g(x) in the form  $ax^4 + bx^3 + cx^2 + dx + e$ , where a, b, c, d,  $e \in \mathbb{Z}$ .
  - (b) Hence, find the sum of the roots of the equation g(x) = 0.

[2]

[5]



7. Let 
$$f(x) = \frac{2x^3 - 5x^2 - 37}{x + 37}$$
, where  $x > 1$ . Solve the inequality  $1 + f(|x|) \le |x|$ . [5]



Prove by mathematical induction that  $\binom{2}{2} + \binom{3}{2} + \ldots + \binom{n}{2} = \frac{n(n+1)(n-1)}{6}$ , 8.  $n \in \mathbb{Z}^+$ ,  $n \ge 2$ .

[7]

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**9.** The random variable *X* has the probability density function

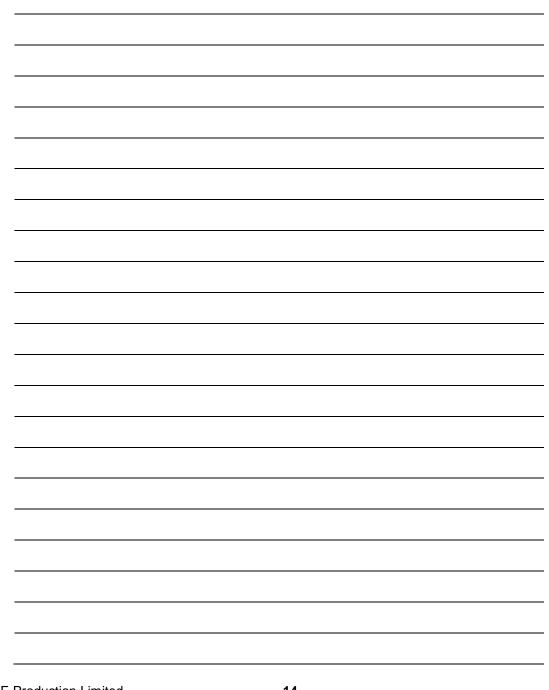
$$f(x) = \frac{1}{e^2 - 1}e^{3-x}, \ 1 \le x \le 3$$

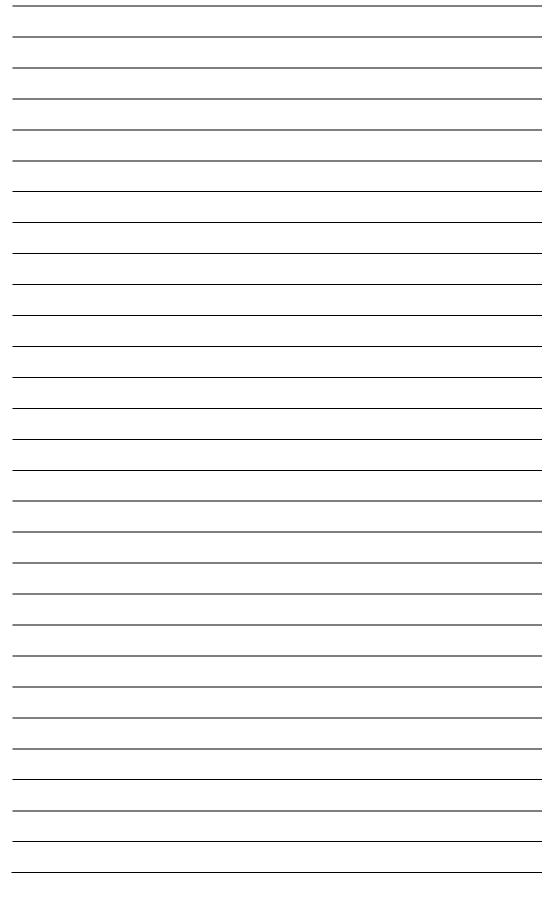
(a) Write down the mode.

[1]

(b) Show that the exact value of the median is  $3 - \ln\left(\frac{e^2 + 1}{2}\right)$ .

[4]





## Section B (54 marks)

**10.** Let  $f(x) = \cos^4 x$ ,  $x \in \mathbb{R}$ .

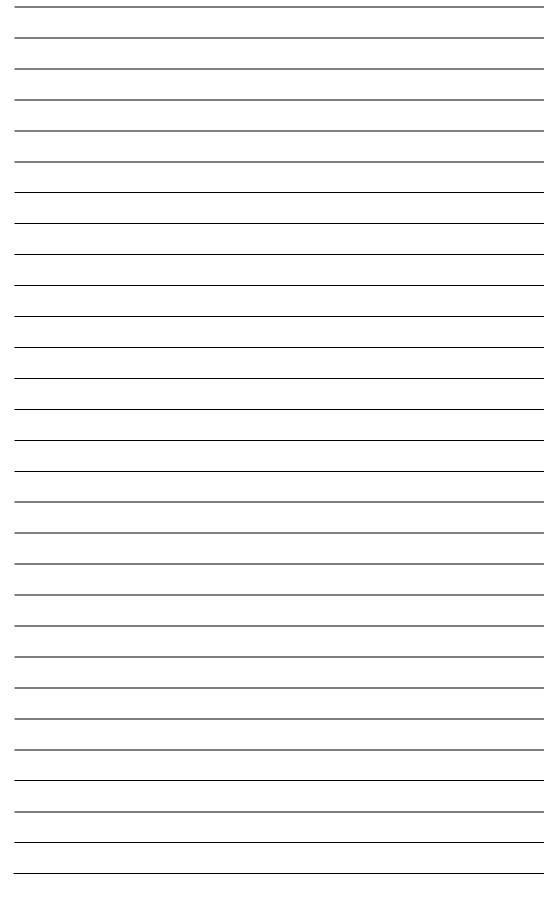
- (a) (i) Write down the range of the function f.
  - (ii) Consider f(x) = 1,  $0 \le x \le 2\pi$ . Find the number of solutions to this equation.
- (b) Find f'(x), giving your answer in the form  $a \sin^p x \cos^q x$  where a, p,  $q \in \mathbb{Z}$ .
- (c) Let  $g(x) = 2\sin x$  for  $0 \le x \le \pi$ . Find the total area of the regions bounded by the graph of y = f(x)g(x) and the *x*-axis.

[7]

[2]

[5]

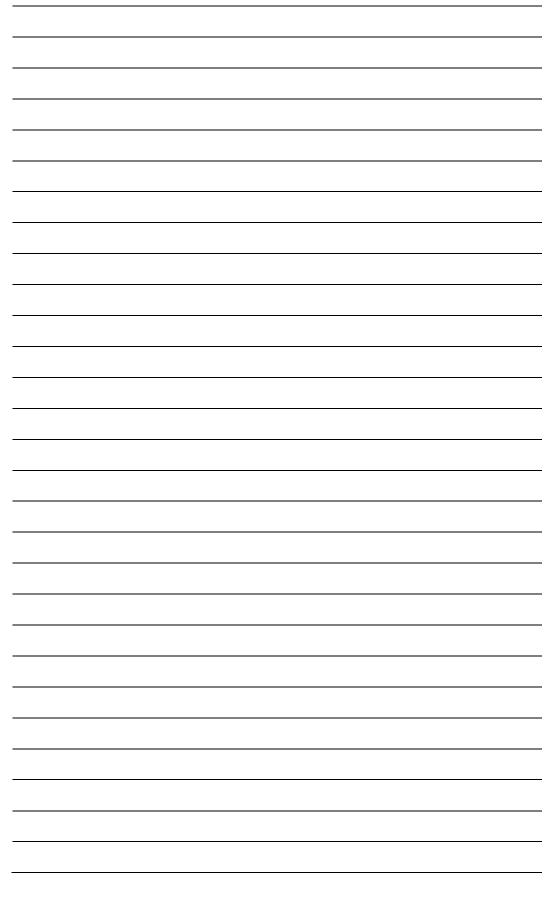




- **11.** A function is defined as  $h(x) = \sin x$ ,  $x \in \mathbb{R}$ .
  - (a) Solve the differential equation  $\frac{dy}{dx} = h(x) \cdot (y+1)$ , y > -1, where y = 0when x = 0, giving the answer in the form y = f(x).

(b) By using the integrating factor approach, show that the solution of the differential equation  $\frac{dy}{dx} = h(x)\sqrt{1-(h(x))^2} \cdot (y+1)$ , where y=0 when x=0, is  $y = e^{\frac{1}{2}\sin^2 x} - 1$ . [12]

[8]



**12.** (a) Solve the equation  $z^6 + 1 = 0$ ,  $z \in \mathbb{C}$ , giving the answers in modulus-argument form.

(b) Hence, solve the equation 
$$z^4 - z^2 + 1 = 0$$
.

Let 
$$\lambda = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$
,  $p = \lambda^3 + \lambda$  and  $q = \lambda^{11} + \lambda^9$ . It is given that  $\lambda + \frac{1}{\lambda} = \sqrt{3}$ .

- (c) (i) Form a quadratic equation of z,  $z \in \mathbb{C}$ , with roots p and q.
  - (ii) Hence, form a quadratic equation of z,  $z \in \mathbb{C}$ , with roots 2pand 2q.

[12]

[4]

[4]