Analysis and Approaches Higher Level
for IBDP Mathematics
Practice Paper Set 1 – Paper 1 (120 Minutes)

Question – Answer Book

Instructions

1. This paper consists of **TWO** sections: A and B.

2. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

3. No calculator is allowed.

4. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.

5. Supplementary answer sheets and graph papers will be supplied on request.

6. Unless otherwise specified, **ALL** working must be clearly shown.

7. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

8. The diagrams in this paper are **NOT** necessarily drawn to scale.

9. Information to be read before you start the exam:

	Marker's Use Only	Examiner's Use Only	
Question Number	**Marks**	**Marks**	**Maximum Mark**
Section A			
1			**6**
2			**5**
3			**6**
4			**8**
5			**7**
6			**7**
7			**5**
8			**7**
9			**5**
Section A Total			**56**
Section B			
10			**14**
11			**20**
12			**20**
Section B Total			**54**
Overall			
Paper 1 Total			**110**

Section A (56 marks)

1. There are 15 items in a data set. The sum of the items is 300.

(a) Find the mean.

[2]

The variance of this data set is 9. Each value in the set is multiplied by -2.

(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance.

(iii) Hence, write down the value of the new standard deviation.

[4]

2. A straight line L_1 passes through the points $(8, 0)$ and $(24, 32)$.

(a) Find the equation of L_1, giving the answer in general form.

[3]

(b) The equation of another straight line, L_2, is given as $x - ay + 2021 = 0$, $a \in \mathbb{R}$. If L_1 and L_2 are perpendicular, find a.

[2]

3. (a) Show that $(2n+1)^2+(2n+3)^2+(2n+5)^2=3(4n^2+12n+11)+2$, where $n\in\mathbb{Z}$.

[3]

(b) Hence, or otherwise, prove that the sum of the squares of any three consecutive odd numbers is greater than a multiple of 3 by 2.

[3]

4. Let $f(x) = px^3 + qx^2 - 2x + 1$. At $x = 1$, the slope of the normal of the curve of f is $-\frac{1}{15}$. It is given that $f^{-1}(41) = 2$, find the value of p and of q.

[8]

5. The graph of f is given by $f(t) = a\sin b(t+2.5)+d$, $a>0$, $t \geq 0$.

When $t=2$, there is a maximum value of 37, at P. When $t=11$, there is a minimum value of -5. The graph of f is strictly decreasing at $2<t<11$.

(a) Show that $f(t) = 21\sin\frac{\pi}{9}(t+2.5)+16$.

[5]

The graph of f is then transformed to the graph of g by a horizontal stretch of scale factor 3, followed by a translation of $\begin{pmatrix}17\\8\end{pmatrix}$. Let P' be the image of P.

(b) Find the coordinates of P'.

[2]

6. Consider the function $f(x) = 4x^4 + 3x^2 - 1,\ x \in \mathbb{R}$. The graph of f is translated one unit to the right and then stretched vertically with scale factor 3 to form the function $g(x)$.

(a) Express $g(x)$ in the form $ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, $e \in \mathbb{Z}$.

[5]

(b) Hence, find the sum of the roots of the equation $g(x) = 0$.

[2]

7. Let $f(x)=\dfrac{2x^3-5x^2-37}{x+37}$, where $x>1$. Solve the inequality $1+f(|x|)\le|x|$.

[5]

8. Prove by mathematical induction that $\binom{2}{2}+\binom{3}{2}+\ldots+\binom{n}{2}=\frac{n(n+1)(n-1)}{6}$,

$n \in \mathbb{Z}^+$, $n \geq 2$.

[7]

9. The random variable X has the probability density function

$$f(x) = \frac{1}{e^2 - 1} e^{3-x}, \ 1 \le x \le 3.$$

(a) Write down the mode.

[1]

(b) Show that the exact value of the median is $3 - \ln\left(\frac{e^2 + 1}{2}\right)$.

[4]

Section B (54 marks)

10. Let $f(x) = \cos^4 x$, $x \in \mathbb{R}$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Find the number of solutions to this equation.

[5]

(b) Find $f'(x)$, giving your answer in the form $a\sin^p x\cos^q x$ where a, p, $q \in \mathbb{Z}$.

[2]

(c) Let $g(x) = 2\sin x$ for $0 \le x \le \pi$. Find the total area of the regions bounded by the graph of $y = f(x)g(x)$ and the x-axis.

[7]

11. A function is defined as $h(x) = \sin x$, $x \in \mathbb{R}$.

(a) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = h(x) \cdot (y+1)$, $y > -1$, where $y = 0$ when $x = 0$, giving the answer in the form $y = f(x)$.

[8]

(b) By using the integrating factor approach, show that the solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = h(x)\sqrt{1-(h(x))^2} \cdot (y+1)$, where $y = 0$ when $x = 0$, is $y = e^{\frac{1}{2}\sin^2 x} - 1$.

[12]

12. (a) Solve the equation $z^6 + 1 = 0$, $z \in \mathbb{C}$, giving the answers in modulus-argument form.

[4]

(b) Hence, solve the equation $z^4 - z^2 + 1 = 0$.

[4]

Let $\lambda = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$, $p = \lambda^3 + \lambda$ and $q = \lambda^{11} + \lambda^9$. It is given that $\lambda + \frac{1}{\lambda} = \sqrt{3}$.

(c) (i) Form a quadratic equation of z, $z \in \mathbb{C}$, with roots p and q.

(ii) Hence, form a quadratic equation of z, $z \in \mathbb{C}$, with roots $2p$ and $2q$.

[12]

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