HKDSE Mathematics Compulsory Part 2020 Mock Exam Set 1 Paper 2 Solution

1.	A	(%)	16.	A	(%)	31.	B	(%)
2.	C	(%)	17.	C	(%)	32.	A	(%)
3.	D	(%)	18.	B	(%)	33.	D	(%)
4.	B	(%)	19.	A	(%)	34.	C	(%)
5.	A	(%)	20.	D	(%)	35.	B	(%)
6.	C	(%)	21.	D	(%)	36.	C	(%)
7.	C	(%)	22.	C	(%)	37.	A	(%)
8.	B	(%)	23.	D	(%)	38.	C	(%)
9.	B	(%)	24.	A	(%)	39.	B	(%)
10.	D	(%)	25.	A	(%)	40.	A	(%)
11.	A	(%)	26.	B	(%)	41.	D	(%)
12.	D	(%)	27.	C	(%)	42.	A	(%)
13.	B	(%)	28.	C	(%)	43.	A	(%)
14.	D	(%)	29.	B	(%)	44.	B	(%)
15.	C	(%)	30.	D	(%)	45.	D	(%)

1. A

$$p = \frac{q-2r}{q+2r}$$
$$p(q+2r) = q-2r$$
$$pq + 2pr = q-2r$$
$$2pr + 2r = q - pq$$
$$2r(p+1) = q(1-p)$$
$$r = \frac{q(1-p)}{2(p+1)}$$

2. C

$$\frac{81^{4y+1}}{27^{7y-2}}$$
$$= \frac{(3^4)^{4y+1}}{(3^3)^{7y-2}}$$
$$= \frac{3^{16y+4}}{3^{21y-6}}$$
$$= 3^{(16y+4)-(21y-6)}$$
$$= 3^{10-5y}$$

3. D

0.007772019 = 0.007772 (*correct to 4 significant figures*)

4. B

$$4x - 16y - x^2 + 16y^2$$
$$= 4(x-4y) - (x^2 - 16y^2)$$
$$= 4(x-4y) - (x+4y)(x-4y)$$
$$= (x-4y)\left[4 - (x+4y)\right]$$
$$= (x-4y)(4-x-4y)$$

5. A

$$\mathrm{f}(3-k)$$
$$= 1 - 2(3-k) - 4(3-k)^2$$
$$= 1 - 6 + 2k - 4(9 - 6k + k^2)$$
$$= 1 - 6 + 2k - 36 + 24k - 4k^2$$
$$= -4k^2 + 26k - 41$$

6. C

$y=(3-x)^2$

$y=(x-3)^2$

$a=1$

$\therefore$ The graph opens upwards.

$\therefore$ A is incorrect.

$0=(x-3)^2$

$x=3$

$\therefore$ There is only one x-intercept.

$\therefore$ B is incorrect.

$y=(x-3)^2$

$h=3$ and $k=0$

$\therefore$ The coordinates of the vertex are $(3, 0)$.

$\therefore$ C is correct.

$y=(0-3)^2$

$y=9$

$\therefore$ The y-intercept of the graph is 9.

$\therefore$ D is incorrect.

7. C

$-30<-2(1-x)$ or $1-x\geq 3$

$-30<-2+2x$ or $-x\geq 2$

$-28<2x$ or $x\leq -2$

$x>-14$ or $x\leq -2$

$\therefore$ The solutions are all real numbers.

8. B

$$\frac{2}{3x-4}-\frac{1}{3x+4}$$

$$=\frac{2(3x+4)-(3x-4)}{(3x+4)(3x-4)}$$

$$=\frac{6x+8-3x+4}{9x^2-16}$$

$$=\frac{3x+12}{9x^2-16}$$

9. B

Amount of interest

$$=80000\left(1+\frac{1.5\%}{12}\right)^{4\times 12}-80000$$

$$=\$4944 \quad (\textit{Correct to the nearest dollar})$$

10. D

$$\text{p}(1)=0$$

$$(1+1)^5-m(1)^2-n=0$$

$$32-m-n=0$$
$$m=32-n$$
$$\text{p}\left(-\frac{1}{2}\right)$$

$$=\left(1-\frac{1}{2}\right)^5-(32-n)\left(-\frac{1}{2}\right)^2-n$$

$$=\frac{1}{32}-\frac{1}{4}(32-n)-n$$

$$=\frac{1}{32}-8+\frac{1}{4}n-n$$

$$=-\frac{255}{32}-\frac{3}{4}n$$

11. A

$$\frac{9}{8a}=\frac{7}{6b}$$
$$54b=56a$$
$$a=\frac{27}{28}b$$
$$\frac{7}{6b}=\frac{5}{4c}$$
$$28c=30b$$
$$c=\frac{15}{14}b$$
$$(a+b):(2b+c)$$
$$=\left(\frac{27}{28}b+b\right):\left(2b+\frac{15}{14}b\right)$$
$$=\frac{55}{28}b:\frac{43}{14}b$$
$$=55:86$$

12. D

$$x_3=2x_1+x_2$$
$$x_3=2(5)+x_2$$
$$x_2=x_3-10$$
$$x_4=2x_2+x_3$$
$$34=2(x_3-10)+x_3$$
$$34=2x_3-20+x_3$$
$$54=3x_3$$
$$x_3=18$$

13. B

The actual length of the square

$=\sqrt{25}$

$=5$ m

$=500$ cm

The side of the square on the map

$=500\div250$

$=2$ cm

14. D

Let $a=\frac{kb^2}{c}$, where k is a non-zero constant.

The new value of a

$=\frac{k(b(1+40\%))^2}{c(1-20\%)}$

$=\frac{k(1.4b)^2}{0.8c}$

$=\frac{1.96kb^2}{0.8c}$

$=\frac{2.45kb^2}{c}$

$=2.45a$

Percentage increase

$=\frac{2.45a-a}{a}\times 100\%$

$=145\%$

15. C

BC^2+AC^2

$=40^2+9^2$

$=1681$

$=41^2$

$=AB^2$

$\therefore \Delta ABC$ is a right-angled triangle, with $\angle ACB=90°$ (*Converse of Pyth. Theorem*)

$\tan A:\cos B:\sin C$

$=\frac{40}{9}:\frac{40}{41}:1$

$=1640:360:369$

16. A

The length of a side of a regular 6-sided polygon

$= 84 \div 6$

$= 14 \text{ cm}$

Note that a regular 6-sided polygon consists of 6 identical equilateral triangles.

Let $h \text{ cm}$ be the height of an equilateral triangle.

$h^2 + 7^2 = 14^2$

$h = \sqrt{147}$

The area of a regular 6-sided polygon

$= (6)\left(\frac{(14)(\sqrt{147})}{2}\right)$

$= 509 \text{ cm}^2$ (*3 s.f.*)

17. C

Let $r \text{ cm}$ be the radius of the upper part.

$\frac{r}{84 - 48} = \frac{35}{84}$

$r = 15$

The curved surface area of the frustum

$= \pi(35)(\sqrt{35^2 + 84^2}) - \pi(15)(\sqrt{15^2 + 36^2})$

$= 2600\pi \text{ cm}^2$

18. B

Let h be the height of the cylinder.

$\frac{h}{r} = \frac{4}{3}$

$h = \frac{4}{3}r$

The ratio of the curved surface area of the cylinder to that of the sphere

$= 2\pi r\left(\frac{4}{3}r\right) : 4\pi(3r)^2$

$= \frac{8}{3}\pi r^2 : 36\pi r^2$

$= 2 : 27$

19. A

$$\frac{\text{Area of } \Delta AFC}{\text{Area of } \Delta CDF} = \frac{3}{2}$$

$$\text{Area of } \Delta AFC = 48 \text{ cm}^2$$

$$\frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta CDF} = \left(\frac{3}{2}\right)^2$$

$$\text{Area of } \Delta AEF = 72 \text{ cm}^2$$

$$\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta AEF} = \frac{2}{3}$$

$$\text{Area of } \Delta DEF = 48 \text{ cm}^2$$

Area of the parallelogram

$= 2(\text{Area of } \Delta ADE)$

$= 2(72 + 48)$

$= 240 \text{ cm}^2$

Area of ΔABC

$= 240 - 32 - 48 - 72 - 48$

$= 40 \text{ cm}^2$

$\therefore BC : CD$

$= \text{Area of } \Delta ABC : \text{Area of } \Delta ACD$

$= 40 : (48 + 32)$

$= 1 : 2$

20. D

Note that ΔOAB is an isosceles triangle.

$\angle OAB + \angle OBA + \angle AOB = 180°$

$2\angle OAB + 30° = 180°$

$\angle OAB = 75°$

The compass bearing of *B* from *A*

$= \text{S}(75° - 30°)\text{E}$

$= \text{S}45°\text{E}$

21. D

$$\left[1-\sin(270°+\theta)\right]\left[1-\cos(720°-\theta)\right]$$

$$=\left[1-(-\cos\theta)\right]\left[1-\cos(360°-\theta)\right]$$

$=(1+\cos\theta)(1-\cos\theta)$

$=1-\cos^2\theta$

$=\sin^2\theta$

22. C

Let $x°$ be the size of an exterior angle.

$x°+11x°=180°$

$12x°=180°$

$x=15$

The number of vertices

$=\dfrac{360°}{15°}$

$=24$

$\therefore$ I is correct.

The size of an interior angle

$=180°-15°$

$=165°$

$\neq 150°$

$\therefore$ II is incorrect.

The number of folds of rotational symmetry of the polygon is equal to its number of sides, which is 24.

$\therefore$ III is correct.

23. D

$\angle AOD$

$= 2\angle ABD$

$= 2(65°)$

$= 130°$

Reflex $\angle AOD$

$= 360° - 130°$

$= 230°$

$\neq 210°$

$\therefore$ I is incorrect.

$\angle OAB + \angle ABD + \angle OBD + \text{reflex}\angle AOD = 360°$

$\angle OAB + 65° + 35° + 230° = 360°$

$\angle OAB = 30°$

$\therefore$ II is correct.

$\because OA = OD$

$\therefore \Delta OAD$ is an isosceles triangle.

$\angle OAD$

$= \dfrac{180° - 130°}{2}$

$= 25°$

$\angle BCD + \angle BAD = 180°$

$\angle BCD + 30° + 25° = 180°$

$\angle BCD = 125°$

$\therefore$ III is correct.

24. A

The polar coordinates of the image of P

$= (10, 150°)$

$x = 10\cos 150°$

$x = -5\sqrt{3}$

$y = 10\sin 150°$

$y = 5$

Therefore, the required coordinates are $(-5\sqrt{3}, 5)$.

25. A

Let $P:(x, y)$.

$AP = AB$

$\sqrt{(x-3)^2+(y-4)^2} = \sqrt{(11-3)^2+(10-4)^2}$

$(x-3)^2+(y-4)^2 = 100$

$x^2-6x+9+y^2-8y+16=100$

$x^2+y^2-6x-8y-75=0$

26. B

The required slope

$= -1 \div \dfrac{27-22}{12-22}$

$= -1 \div \dfrac{5}{-10}$

$= 2$

$2x+3(0)+6=0$

$2x+6=0$

$x=-3$

The required point is $(-3, 0)$.

The equation:

$y-0=2(x-(-3))$

$y=2x+6$

$2x-y+6=0$

27. C

The median height of the girls is 161 cm.

$\therefore \dfrac{158+x}{2}=161$

$x=164$

The mean height of the girls

$= \dfrac{138+152+178+158+170+164}{6}$

$=160$ cm

28. C

The minimum value is 40.

The lower quartile

$= \frac{59+61}{2}$

$= 60$

The median

$= \frac{68+68}{2}$

$= 68$

The upper quartile

$= \frac{84+86}{2}$

$= 85$

The maximum value is 89.

Therefore, the right half of the box should be relatively longer than the left half, with a short whisker on the right and a longer whisker on the left.

29. B

The required probability

$= \left(\frac{2}{5}\right)\left(\frac{4}{4}\right) + \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)$

$= \frac{7}{10}$

30. D

The multiples of 15 between 100 and 199 inclusive are 105, 120, 135, 150, 165, 180 and 195.

The required probability

$= \frac{100-7}{100}$

$= \frac{93}{100}$

31. B

$x^4 - x^2$
$= x^2(x^2 - 1)$
$= x^2(x+1)(x-1)$
$x^3 + 3x^2 + 2x$
$= x(x^2 + 3x + 2)$
$= x(x+1)(x+2)$
$x^2y + x^3y$
$= x^2y(1+x)$
$\therefore \text{H.C.F.} = x(1+x)$

32. A

$\log 0.538^{77127}$

$= 77127 \log 0.538$
$= -20763.95542$

$\log 0.654^{86834}$

$= 86834 \log 0.654$
$= -16014.1218$

$\log 0.765^{98100}$

$= 98100 \log 0.765$
$= -11412.81321$

$\log 0.823^{115392}$

$= 115392 \log 0.823$
$= -9762.182215$

As $\log 0.538^{77127}$ is the smallest, 0.538^{77127} is the smallest.

33. D

$3^{2y} + \log_7 x = 10$

$(3^y)^2 + \log_7 x = 10$

$\therefore (\log_7 x + 2)^2 + \log_7 x = 10$

$(\log_7 x)^2 + 4\log_7 x + 4 + \log_7 x = 10$

$(\log_7 x)^2 + 5\log_7 x - 6 = 0$

$(\log_7 x + 6)(\log_7 x - 1) = 0$

$\log_7 x = -6$ or $\log_7 x = 1$

$x = 7^{-6}$ or $x = 7^1$

When $x = 7^{-6}$,

$3^y = -6 + 2$

$3^y = -4$ (*Rejected*)

When $x = 7$,

$3^y = 1 + 2$

$3^y = 3$

$y = 1$

$\therefore x + y$

$= 7 + 1$

$= 8$

34. C

$3 \times 16^2 + 2 \times 4^2 + 1 \times 2^2$

$= 3 \times (2^4)^2 + 2 \times (2^2)^2 + 1 \times 2^2$

$= 3 \times 2^8 + 2 \times 2^4 + 1 \times 2^2$

$= (2+1) \times 2^8 + 2^5 + 2^2$

$= 2^9 + 2^8 + 2^5 + 2^2$

$= 1100100100_2$

35. B

$\frac{1}{h} = h - 2$

$1 = h^2 - 2h$

$h^2 - 2h - 1 = 0$

$k = \frac{k^2 - 1}{2}$

$2k = k^2 - 1$

$k^2 - 2k - 1 = 0$

$\therefore h$ and k are the roots of the quadratic equation $x^2 - 2x - 1 = 0$.

$h + k$

$= -\frac{-2}{1}$

$= 2$

hk

$= \frac{-1}{1}$

$= -1$

$\therefore (h-2)(k-2)$

$= hk - 2h - 2k + 4$

$= hk - 2(h+k) + 4$

$= -1 - 2(2) + 4$

$= -1$

36. C

$\frac{i^2 + 9}{i - 3}$

$= \frac{(-1+9)(-i-3)}{(i-3)(-i-3)}$

$= \frac{8(-3-i)}{(-3+i)(-3-i)}$

$= \frac{-24-8i}{(-3)^2 - i^2}$

$= \frac{-24-8i}{10}$

$= -\frac{12}{5} - \frac{4}{5}i$

37. A

$\frac{4\pi}{2\pi}=2$ and $\frac{8\pi}{4\pi}=2$

$\therefore$ I is correct.

$\frac{4^{\pi}}{2^{\pi}}$

$=\left(\frac{4}{2}\right)^{\pi}$

$=2^{\pi}$

$\frac{8^{\pi}}{4^{\pi}}$

$=\left(\frac{8}{4}\right)^{\pi}$

$=2^{\pi}$

$\therefore$ II is correct.

$\frac{\log_4 \pi}{\log_2 \pi}$

$=\frac{\frac{\log \pi}{\log 4}}{\frac{\log \pi}{\log 2}}$

$=\frac{\log 2}{\log 4}$

$=\frac{1}{2}$

$\frac{\log_8 \pi}{\log_4 \pi}$

$=\frac{\frac{\log \pi}{\log 8}}{\frac{\log \pi}{\log 4}}$

$=\frac{\log 4}{\log 8}$

$=\frac{2}{3}$

$\neq \frac{1}{2}$

$\therefore$ III is incorrect.

38. C

$$\begin{cases} 4+9=13\geq 10 \\ 4+4(9)=40\leq 40 \\ 4(4)+9=25\leq 40 \end{cases}$$

Thus, $(4, 9)$ lies in D.

$\therefore$ I is correct.

Note that $(0,10)$, $(8,8)$ and $(10, 0)$ are the vertices of D.

At $(0,10)$: $2x+y=2(0)+10=10$

At $(8,8)$: $2x+y=2(8)+8=24$

At $(10, 0)$: $2x+y=2(10)+20=0$

Thus, the greatest value is not 16.

$\therefore$ II is incorrect.

The distance between $(0,10)$ and $(8,8)$

$=\sqrt{(8-0)^2+(8-10)^2}$

$=\sqrt{68}$

The distance between $(10, 0)$ and $(8,8)$

$=\sqrt{(8-10)^2+(8-0)^2}$

$=\sqrt{68}$

The distance between $(0,10)$ and $(10, 0)$

$=\sqrt{(10-0)^2+(0-10)^2}$

$=\sqrt{200}$

Thus, D is an isosceles triangle.

$\therefore$ III is correct.

39. B

$\cos x = 11\cos^2 x$

$11\cos^2 x - \cos x = 0$

$\cos x(11\cos x - 1) = 0$

$\cos x = 0$ or $11\cos x - 1 = 0$

$\cos x = 0$ or $\cos x = \frac{1}{11}$

$x = 90°$ (*Rejected*), $270°$ or $x = 84.78409143°$ (*Rejected*) or $360° - 84.78409143°$

$x = 270°$ or $275.2159086°$

Thus, there are two roots.

40. A

$\angle OAB = \angle OAC$, $\angle OBA = \angle OBC$ and $\angle OCA = \angle OCB$ (*In-centre*)

Also, $\angle OAB = \angle OBA$, $\angle OBC = \angle OCB$ and $\angle OAC = \angle OCA$ (*Radii*)

Thus, $\angle OAB = \angle OAC = \angle OBA = \angle OBC = \angle OCA = \angle OCB$

$\angle OAB + \angle OAC + \angle OBA + \angle OBC + \angle OCA + \angle OCB = 180°$ ($\angle$ *sum of* Δ)

$6\angle OAB = 180°$

$\angle OAB = 30°$

$\angle OAB + \angle TAB = 90°$ (*Tangent* $\perp$ *radius*)

$30° + \angle TAB = 90°$

$\angle TAB = 60°$

Similarly, $\angle TBA = 60°$.

$\angle ATB + \angle TAB + \angle TBA = 180°$ ($\angle$ *sum of* Δ)

$\angle ATB + 60° + 60° = 180°$

$\angle ATB = 60°$

$\therefore \angle ATB + \angle ACB$

$= 60° + 30° + 30°$

$= 120°$

41. D

Consider the formulas for the volume V and the total surface area A of a tetrahedron with given height h:

$$\begin{cases} V = \dfrac{\sqrt{3}}{8} h^3 \\ A = \dfrac{3\sqrt{3}}{2} h^2 \end{cases}$$

$$\frac{\sqrt{3}}{8} h^3 = \sqrt{3}$$

$$h^3 = 8$$

$$h = 2$$

The total surface area

$$= \frac{3\sqrt{3}}{2}(2)^2$$

$$= 6\sqrt{3} \text{ cm}^2$$

42. A

$$5x - 12y - 50 = 0$$

$$12y = 5x - 50$$

$$y = \frac{5x - 50}{12}$$

$$x^2 + y^2 - 10x - 24y + k = 0$$

$$\therefore x^2 + \left(\frac{5x-50}{12}\right)^2 - 10x - 24\left(\frac{5x-50}{12}\right) + k = 0$$

$$\therefore x^2 + \frac{25}{144}x^2 - \frac{125}{36}x + \frac{625}{36} - 10x - 10x + 100 + k = 0$$

$$\frac{169}{144}x^2 - \frac{845}{36}x + \frac{4225}{36} + k = 0$$

$$169x^2 - 3380x + 16900 + 144k = 0$$

As the circle and the straight line do not intersect, $\Delta < 0$.

$$(-3380)^2 - 4(169)(16900 + 144k) < 0$$

$$11424400 - 11424400 - 97344k < 0$$

$$-97344k < 0$$

$$k > 0$$

43. A

Number of different teams

$= C_2^6 \times C_4^{14} + C_3^6 \times C_3^{14} + C_4^6 \times C_2^{14} + C_5^6 \times C_1^{14} + C_6^6 \times C_0^{14}$

$= 23745$

44. B

The required probability

$= \frac{1}{2} \times \frac{2}{6} + \frac{1}{2} \times \frac{3}{8}$

$= \frac{17}{48}$

45. D

Let d be the common difference of the arithmetic sequences.

Note that B is created after every data in A is added by $50d$.

The mean of B

$= 50d +$ The mean of A

$\neq$ The mean of A

$\therefore$ I is incorrect.

Note that the range and the standard deviation is unchanged after $50d$ is added to each data.

Thus, the range of A and the range of B are the same.

$\therefore$ II is correct.

Similarly, the standard deviation of A and the standard deviation of B are the same.

$\therefore$ III is correct.