

Chapter 23 Solution

Exercise 96

1. (a) H_0 : The number of heads follows the assigned distribution. A1 N1 [1]
- (b) 10 A1 N1 [1]
- (c) 3 A1 N1 [1]
- (d) $\chi^2_{calc} = 6.933333333$ (A1) for correct value
 $\chi^2_{calc} = 6.93$ A1 N2 [2]
- (e) The null hypothesis is not rejected. A1
As $\chi^2_{calc} < 7.815$. A1 N2 [2]
2. (a) H_0 : The last digits are evenly distributed. A1 N1 [1]
- (b) 10 A1 N1 [1]
- (c) 9 A1 N1 [1]
- (d) 15.8 A2 N2 [2]
- (e) The null hypothesis is not rejected. A1
As $\chi^2_{calc} < 16.919$. A1 N2 [2]

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| 3. | (a) | H_0 : The number of emails received on each day follows the assigned distribution. | A1 | N1 | [1] |
| | (b) | $a = 4, b = 5$ | A2 | N2 | [2] |
| | (c) | 5 | A1 | N1 | [1] |
| | (d) | p -value = 0.0907073916
p -value = 0.0907 | (A1) for correct value
A1 | N2 | [2] |
| | (e) | The null hypothesis is not rejected.
As p -value > 0.05 . | A1
A1 | N2 | [2] |
| 4. | (a) | H_0 : The outcomes follows the assigned distribution. | A1 | N1 | [1] |
| | (b) | 30 | A1 | N1 | [1] |
| | (c) | 6 | A1 | N1 | [1] |
| | (d) | p -value = 0.00004963145972
p -value = 0.0000496 | (A1) for correct value
A1 | N2 | [2] |
| | (e) | The null hypothesis is rejected.
As p -value < 0.01 . | A1
A1 | N2 | [2] |

Exercise 97

1. (a) (i) H_0 : The examination results and the corresponding sections are independent. A1 N1
- (ii) H_1 : The examination results and the corresponding sections are not independent. A1 N1 [2]
- (b) 2 A1 N1 [1]
- (c) $\chi^2_{calc} = 17.41714286$ (A1) for correct value
 $\chi^2_{calc} = 17.4$ A1 N2 [2]
- (d) The null hypothesis is rejected. A1
 As $\chi^2_{calc} > 5.991$. A1 N2 [2]
2. (a) (i) H_0 : The nationality of children and the choices of their most favourite fruits are independent. A1 N1
- (ii) H_1 : The nationality of children and the choices of their most favourite fruits are not independent. A1 N1 [2]
- (b) 6 A1 N1 [1]
- (c) $p\text{-value} = 4.56640566 \times 10^{-10}$ (A1) for correct value
 $p\text{-value} = 4.57 \times 10^{-10}$ A1 N2 [2]
- (d) The null hypothesis is rejected. A1
 As $p\text{-value} < 0.01$. A1 N2 [2]

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| 3. | (a) | H_0 : The age of adults and their preferences are independent. | A1 | N1 | [1] |
| | (b) | $x = 13, y = 39$ | A2 | N2 | [2] |
| | (c) | 4 | A1 | N1 | [1] |
| | (d) | $p\text{-value} = 4.702920665 \times 10^{-7}$
$p\text{-value} = 4.70 \times 10^{-7}$ | (A1) for correct value
A1 | N2 | [2] |
| | (e) | The null hypothesis is rejected.
As $p\text{-value} < 0.05$. | A1
A1 | N2 | [2] |
| 4. | (a) | H_0 : The age of staffs and their number of investment bank accounts are independent. | A1 | N1 | [1] |
| | (b) | $x = 5, y = 4$ | A2 | N2 | [2] |
| | (c) | 8 | A1 | N1 | [1] |
| | (d) | $p\text{-value} = 0.3261508802$
$p\text{-value} = 0.326$ | (A1) for correct value
A1 | N2 | [2] |
| | (e) | The null hypothesis is not rejected.
As $p\text{-value} > 0.05$. | A1
A1 | N2 | [2] |

Exercise 98

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|----|-----|---|------------------------------|----|-----|
| 1. | (a) | The lengths of fishes are normally distributed. | A1 | N1 | [1] |
| | (b) | (i) $H_0: \mu_1 = \mu_2$ | A1 | N1 | |
| | | (ii) $H_1: \mu_1 \neq \mu_2$ | A1 | N1 | [2] |
| | (c) | p -value = 0.0799994402
p -value = 0.0800 | (A1) for correct value
A1 | N2 | [2] |
| | (d) | The null hypothesis is not rejected.
As p -value > 0.05. | A1
A1 | N2 | [2] |
| 2. | (a) | The volumes of bottles of milk are normally distributed. | A1 | N1 | [1] |
| | (b) | (i) $H_0: \mu_D = \mu_L$ | A1 | N1 | |
| | | (ii) $H_1: \mu_D > \mu_L$ | A1 | N1 | [2] |
| | (c) | p -value = 0.0300468645
p -value = 0.0300 | (A1) for correct value
A1 | N2 | [2] |
| | (d) | The null hypothesis is rejected.
As p -value < 0.05. | A1
A1 | N2 | [2] |
| 3. | (a) | $H_1: \mu_1 < \mu_2$ | A1 | N1 | [1] |
| | (b) | p -value = 0.0276305613
p -value = 0.0276 | (A1) for correct value
A1 | N2 | [2] |
| | (c) | -2.09 | A1 | N1 | [1] |
| | (d) | The null hypothesis is rejected.
As p -value < 0.05. | A1
A1 | N2 | [2] |

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| 4. | (a) | $H_1: \mu_1 \neq \mu_2$ | A1 | N1 | [1] |
| | (b) | $p\text{-value} = 0.9063680224$
$p\text{-value} = 0.906$ | (A1) for correct value | A1 | N2 |
| | | | | | [2] |
| | (c) | -0.120 | A1 | N1 | [1] |
| | (d) | The null hypothesis is not rejected.
As $p\text{-value} > 0.1$. | A1 | A1 | N2 |
| | | | | | [2] |

Exercise 99

1. (a) (i) H_0 : The punctuality of buses and the locations of bus stops are independent. A1 N1
- (ii) H_1 : The punctuality of buses and the locations of bus stops are not independent. A1 N1 [2]
- (b) 3 A1 N1 [1]
- (c) $\chi^2_{calc} = 6.407563025$ (A1) for correct value
 $\chi^2_{calc} = 6.41$ A1 N2 [2]
- (d) The null hypothesis is rejected. A1
 As $\chi^2_{calc} > 6.251$. A1 N2 [2]
- (e) (i) The required probability

$$= \frac{20}{100}$$

$$= \frac{1}{5}$$
 (A1) for correct formula A1 N2
- (ii) The required probability

$$= \frac{1+3+8+3}{100}$$

$$= \frac{3}{20}$$
 (A1) for correct formula A1 N2
- (iii) The required probability

$$= \frac{19}{19+17+20+29}$$

$$= \frac{19}{85}$$
 (A1) for correct formula A1 N2 [6]
- (f) The required probability

$$= \left(\frac{85}{100}\right)\left(\frac{85-1}{100-1}\right)$$

$$= \frac{119}{165}$$
 (A2) for correct formula A1 N3 [3]

2. (a) (i) H_0 : The number of free lunches offered and the positions of staffs are independent. A1 N1
- (ii) H_1 : The number of free lunches offered and the positions of staffs are not independent. A1 N1 [2]
- (b) 9 A1 N1 [1]
- (c) $p\text{-value} = 5.946862571 \times 10^{-7}$ (A1) for correct value
 $p\text{-value} = 5.95 \times 10^{-7}$ A1 N2 [2]
- (d) 46.0 A1 N1 [1]
- (e) The null hypothesis is rejected. A1
As $p\text{-value} < 0.05$. A1 N2 [2]
- (f) (i) The required probability

$$= \frac{4}{100}$$

$$= \frac{1}{25}$$
 (A1) for correct formula A1 N2
- (ii) The required probability

$$= \frac{2+10+2+2+1+10+3+2}{100}$$

$$= \frac{8}{25}$$
 (A1) for correct formula A1 N2
- (iii) The required probability

$$= \frac{2}{2+10+2+2}$$

$$= \frac{1}{8}$$
 (A1) for correct formula A1 N2 [6]
- (g) The required probability

$$= \left(\frac{68}{100}\right)\left(\frac{68-1}{100-1}\right)$$

$$= \frac{1139}{2475}$$
 (A2) for correct formula A1 N3 [3]

3.	(a)	(i)	8	A1	N1	
		(ii)	5.5	A1	N1	
		(iii)	$1 \leq X \leq 5$	A1	N1	[3]
	(b)	(i)	$a = 100, b = 60$	A2	N2	
		(ii)	6.5	A1	N1	
		(iii)	3.91	A1	N1	[4]
	(c)	The required probability				
			$= \frac{15+13}{200}$	(A1) for correct formula		
			$= \frac{7}{50}$	A1	N2	
						[2]
	(d)	(i)	H_0 : The number of dolls owned by a female student and her nationality are independent.	A1	N1	
		(ii)	H_1 : The number of dolls owned by a female student and her nationality are not independent.	A1	N1	[2]
	(e)		4	A1	N1	[1]
	(f)		p -value = 0.0623160411	(A1) for correct value		
			p -value = 0.0623	A1	N2	[2]
	(g)		8.95	A1	N1	[1]
	(h)		The null hypothesis is not rejected.	A1		
			As p -value > 0.05 .	A1	N2	[2]

4.	(a)	(i)	25.5	A1	N1	
		(ii)	10.5	A1	N1	
		(iii)	$1 \leq X \leq 10$	A1	N1	[3]
	(b)	(i)	$a = 30, b = 146, n = 500$	A3	N3	
		(ii)	14.42	A1	N1	
		(iii)	8.25	A1	N1	[5]
	(c)	The required probability				
			$= \frac{75 + 125 + 65 + 15}{154 + 146}$	(A2)	for correct formula	
			$= \frac{14}{15}$	A1	N3	
						[3]
	(d)	(i)	H_0 : The age of an interviewee and the number of yogurt parfait cups ate are independent.	A1	N1	
		(ii)	H_1 : The age of an interviewee and the number of yogurt parfait cups ate are not independent.	A1	N1	[2]
	(e)		6	A1	N1	[1]
	(f)		$p\text{-value} = 1.443177288 \times 10^{-31}$	(A1)	for correct value	
			$p\text{-value} = 1.44 \times 10^{-31}$	A1	N2	[2]
	(g)		158	A1	N1	[1]
	(h)		The null hypothesis is rejected.	A1		
			As $p\text{-value} < 0.01$.	A1	N2	[2]

Exercise 100

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|----|-----|------|--|--------------------------------|----|-----|
| 1. | (a) | (i) | $H_0: \mu_1 = \mu_2$ | A1 | N1 | |
| | | (ii) | $H_1: \mu_1 < \mu_2$ | A1 | N1 | [2] |
| | (b) | | $p\text{-value} = 0.365277312$
$p\text{-value} = 0.365$ | (A1) for correct value
A1 | N2 | [2] |
| | (c) | | -0.350 | A1 | N1 | [1] |
| | (d) | | The null hypothesis is not rejected.
As $p\text{-value} > 0.1$. | A1
A1 | N2 | [2] |
| | (e) | | The required probability
$= \frac{3}{3+5}$
$= \frac{3}{8}$ | (A1) for correct formula
A1 | N2 | [2] |
| | (f) | (i) | The required probability
$= \left(\frac{5}{9}\right)\left(\frac{6}{10}\right)$
$= \frac{1}{3}$ | (A1) for correct formula
A1 | N2 | [2] |
| | | (ii) | The required probability
$= 1 - \frac{1}{3}$
$= \frac{2}{3}$ | (A1) for correct formula
A1 | N2 | [4] |

2. (a) The number of times for a person to smoke in a day are normally distributed. A1 N1 [1]
- (b) $H_1: \mu_1 > \mu_2$ A1 N1 [1]
- (c) $p\text{-value} = 0.0055603914$ (A1) for correct value
 $p\text{-value} = 0.00556$ A1 N2 [2]
- (d) 3.11 A1 N1 [1]
- (e) The null hypothesis is rejected. A1
 As $p\text{-value} < 0.01$. A1 N2 [2]
- (f) (i) The required probability

$$= \frac{3}{12}$$

$$= \frac{1}{4}$$
 (A1) for correct formula
 A1 N2
- (ii) The required probability

$$= \frac{3}{6+3}$$

$$= \frac{1}{3}$$
 (A1) for correct formula
 A1 N2 [4]
- (g) The required probability

$$= \left(\frac{7}{12}\right)\left(\frac{7-1}{12-1}\right)\left(\frac{7-2}{12-2}\right)$$

$$= \frac{7}{44}$$
 (A2) for correct formula
 A1 N3 [3]

3. (a) The scores in the assessments are normally distributed. A1 N1 [1]
- (b) $H_1: \mu_{A1} \neq \mu_{B1}$ A1 N1 [1]
- (c) $p\text{-value} = 0.0465127021$ (A1) for correct value
 $p\text{-value} = 0.0465$ A1 N2 [2]
- (d) 2.20 A1 N1 [1]
- (e) The null hypothesis is rejected. A1
 As $p\text{-value} < 0.05$. A1 N2 [2]
- (f) (i) $\mu_{A1} < \mu_{A2}$ A1 N1 [2]
- (ii) $p\text{-value} = 0.9003407897$ (A1) for correct value
 As $p\text{-value} > 0.05$, the null hypothesis is not rejected. A2 N3 [4]
- (g) The required probability

$$= \left(\frac{5}{9}\right)\left(\frac{2}{7}\right) + \left(\frac{4}{9}\right)\left(\frac{5}{7}\right)$$
 (A2) for correct formula

$$= \frac{10}{21}$$
 A1 N3 [3]

4. (a) The ball speeds of free kicks are normally distributed. A1 N1 [1]
- (b) $H_1: \mu_{R1} < \mu_{R2}$ A1 N1 [1]
- (c) p -value = 0.0485601919 (A1) for correct value
 p -value = 0.0486 A1 N2 [2]
- (d) -1.77 A1 N1 [1]
- (e) The null hypothesis is rejected. A1
 As p -value < 0.1. A1 N2 [2]
- (f) (i) $\mu_{R2} \neq \mu_{M2}$ A1 N1 [2]
- (ii) p -value = 0.3962892074 (A1) for correct value
 As p -value > 0.1, the null hypothesis is not rejected. A2 N3 [4]
- (g) The required probability

$$= \binom{2}{8} \binom{2}{9} + \binom{2}{8} \binom{4}{9}$$

$$= \frac{1}{6}$$
 (A2) for correct formula A1 N3 [3]