Analysis and Approaches Standard Level for IBDP Mathematics
Practice Paper Set 1 – Paper 2 (90 Minutes)

Question – Answer Book

Instructions

1. This paper consists of **TWO** sections: A and B.

2. Attempt **ALL** questions. Write your answers in the spaces provided in this Question - Answer Book.

3. A graphic display calculator is needed.

4. You are suggested to prepare a formula booklet of Analysis and Approaches for IBDP Mathematics when attempting the questions.

5. Supplementary answer sheets and graph papers will be supplied on request.

6. Unless otherwise specified, **ALL** working must be clearly shown.

7. Unless otherwise specified, numerical answers should be either **EXACT** or correct to **3 SIGNIFICANT FIGURES**.

8. The diagrams in this paper are **NOT** necessarily drawn to scale.

9. Information to be read before you start the exam:

	Marker's Use Only	**Examiner's Use Only**	
Question Number	**Marks**	**Marks**	**Maximum Mark**
Section A			
1			**6**
2			**7**
3			**7**
4			**7**
5			**6**
6			**7**
Section A Total			**40**
Section B			
7			**12**
8			**14**
9			**14**
Section B Total			**40**
Overall			
Paper 2 Total			**80**

Section A (40 marks)

1. Let $f(x)=3x+7$ and $g(x)=2\sqrt{x}$.

(a) Find $f^{-1}(x)$.

[2]

(b) Find $(f\circ g)(x)$.

[2]

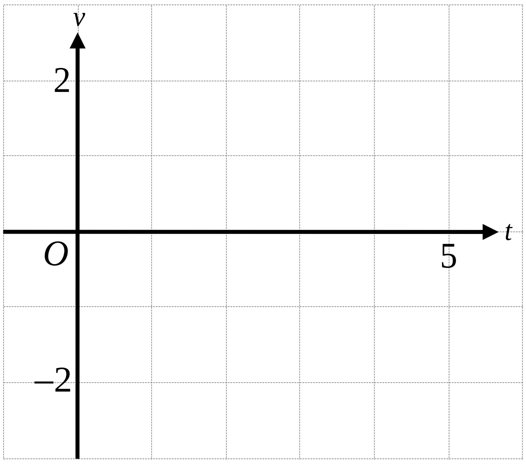
(c) Hence, find $(f\circ g)(529)$.

[2]

2. The velocity $v \text{ ms}^{-1}$ of an object after t seconds is given by
$v(t) = 2.5t - 5.6\sqrt{t} + 2$, for $0 \le t \le 5$.

(a) On the grid below, sketch the graph of v, clearly indicating the minimum point.

[3]



Let d be the distance travelled in the first five seconds.

(b) (i) Write down an expression for d.

(ii) Hence, write down the value of d.

[4]

3. The radius and the vertical height of a metal circular cone are both equal to $18\,\text{cm}$.

(a) Find the volume of the circular cone expressing your answer in the form $a \times 10^k$, $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[3]

16 identical circular cones are to be melted down and remoulded into 27 identical metal hemispheres.

(b) Find the ratio of the radius of the cone to the radius of the hemisphere.

[4]

4. In a geometric series, $u_1 = 4.5$ and $u_2 = 5.4$.

(a) Find the value of r.

[2]

(b) Find the value of S_{12}.

[2]

(c) Find the greatest value of n such that $u_n < 678$.

[3]

5. In the expansion of $2ax(1+3ax^2)^{17}$, the coefficient of the term in x^9 is -385560. Find the value of a.

[6]

6. A population of eagles P_t can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $20P_1 - 17P_0 = 0$. It is given that one decade is equivalent to ten years.

(a) Show that $k = \ln 0.85$.

[2]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} \leq 0.5$.

[5]

Section B (40 marks)

7. The insurance cost of an electric car depends partly on the distance it has travelled. The following table shows the distance and the insurance costs for five electric cars on 1 January 2021.

Travelling distance (x km)	20000	25000	30000	35000	40000
Insurance cost (\$ y)	12000	10800	9600	9000	8500

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) Write down the value of a and of b.

[2]

On 1 January 2021, Hyung Sik buys an electric car which has travelled 32500 km.

(b) Use the regression equation to estimate the insurance cost of Hyung Sik's electric car.

[2]

The insurance cost of an electric car decreases by 2.5% each year.

(c) Calculate the insurance cost of Hyung Sik's electric car after 4 years.

[4]

Hyung Sik will buy an extra electric car when the insurance cost of his original electric car reaches $\$6500$.

(d) Find the year when Hyung Sik buys an extra electric car.

[4]

8. At 8 am on any school day, Stephen walks from his apartment to the school bus stop. The time T minutes needed for Stephen to walk to the school bus stop follows a normal distribution such that $T \sim \text{N}(16, 5^2)$.

There are two school buses which will depart at $8:12\text{ am}$ and $8:24\text{ am}$ respectively, in every school day morning. Stephen will take the first bus if he arrives at or before $8:12\text{ am}$.

(a) Find the probability that Stephen can arrive at the school bus stop before the second school bus departs.

[2]

The time U minutes needed for a school bus to travel from the school bus stop to school follows a normal distribution such that $U \sim \text{N}(\mu, 7^2)$. It is given that U and T are independent, and 99.494% of the school buses take at most 48 minutes to travel from the school bus stop to school.

(b) Find μ.

[3]

In order to be marked as on time, Stephen needs to take any one of the buses and arrive at school by 9 am.

(c) Show that, correct to five significant figures, for all school buses departing at $8:24\text{ am}$, 80.439% of them will arrive at school on time.

[2]

(d) Hence, find the probability that Stephen will not arrive at school on time.

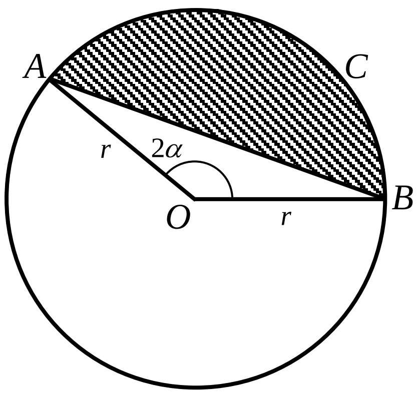
[5]

There are twenty school days in February 2021.

(e) Find the expected number of school days that Stephen will not arrive at school on time in February 2021.

[2]

9. Consider the following circle with centre O and radius r.



The points A, B and C are on the circumference such that $\mathrm{A\hat{O}B} = 2\alpha$, $0 < \alpha < \dfrac{\pi}{2}$.

(a) Show that $\mathrm{AB} = r\sqrt{2(1-\cos 2\alpha)}$.

[2]

Let P be the perimeter of the shaded region.

(b) Show that $P = 2r(\alpha + \sin\alpha)$.

[5]

Consider the function $f(\theta) = \theta + \sin\theta$, $0 < \theta < \dfrac{\pi}{2}$.

(c) Write down the value of θ when

(i) $f(\theta) - 2 = 0$;

(ii) $2f(\theta) - 3 = 0$.

[2]

(d) Hence, find the range of values of α such that P lies exclusively between 1.5 to 2 times the diameter of the circle.

[5]

END OF PAPER