Exercise 2.1

(a) (i) $P(0)=50$ (A1)

(ii) $P^{-1}(70)=5$ (A1)

(b) $P(50)$

$=0.8(50)^2+50$ $P(50)$ (M1)

$=2050$

Thus, the range of P is $50 \le P \le 2050$, $N \in \mathbb{R}$. (A1)

(c) The price of the journey is 370 dollars when the passenger lives 20 kilometres from the airport. (A1)

(d) $P=0.8a^2+50$

$\rightarrow a=0.8P^2+50$ Interchange a and P (M1)

$a-50=0.8P^2$

$1.25a-62.5=P^2$

$P=\sqrt{1.25a-62.5}$

$\therefore P^{-1}(a)=\sqrt{1.25a-62.5}$ (A1)

Exercise 2.2

(a) Let $P = \frac{k}{A}$, where $k \neq 0$. $P = \frac{k}{A}$ (M1)

$15 = \frac{k}{16}$

$k = 240$

$\therefore P = \frac{240}{A}$ (A1)

(b) $\$3$ (A1)

(c) The price of a tetrahedron model of a large surface area will approach $\$0$. (A1)

(d) α

$= \frac{14400}{P^2}$

$= \frac{14400}{\left(\frac{240}{A}\right)^2}$ $\frac{14400}{\left(\frac{240}{A}\right)^2}$ (M1)

$= 0.25A^2$ (A1)

Exercise 2.3

(a) 19 cm^2 (A1)

(b) (i) $x > 18,\ x \in \mathbb{R}$ (A1)

(ii) $A > 0,\ y \in \mathbb{R}$ (A1)

(c) (i) $A = 55$

$\therefore 0.5x^2 - 9.5x + 9 = 55$

$0.5x^2 - 9.5x - 46 = 0$ Correct equation (A1)

By considering the graph of $y = 0.5x^2 - 9.5x - 46$, the horizontal intercept is 23. GDC approach (M1)

$\therefore x = 23$ (A1)

(ii) The length of the longest side

$= \sqrt{(23-18)^2 + (23-1)^2}$

$= \sqrt{509}$ $\sqrt{509}$ (A1)

The required perimeter

$= (23-18) + (23-1) + \sqrt{509}$ The sum of 3 sides (M1)

$= 49.56102835\text{ cm}$

$= 49.6\text{ cm}$ (A1)

Exercise 2.4

(a) $P = P_0 e^{kt}$

$\therefore P_0(1-10\%) = P_0 e^{k(1)}$ — $P_1 = P_0(1-10\%)$ & $t = 1$ (A1)

$0.9 = e^k$

$k = \ln 0.9$ — $b = e^x \Leftrightarrow x = \ln b$ (M1)

$k = -0.1053605157$

$\therefore k = -0.1054$ — (A1)

(b) The growth rate is negative. — (R1)

(c) $0.5P_0 = P_0 e^{-0.1053605157t}$ — Correct equation (A1)

$0.5 = e^{-0.1053605157t}$

$e^{-0.1053605157t} - 0.5 = 0$

By considering the graph of

$y = e^{-0.1053605157t} - 0.5$, the horizontal intercept is

6.5788135. — GDC approach (M1)

$\therefore$ The least number of complete years is 66. — (A1)

(d) t

$= 71 - 18.8\log_{10} 3000$ — $Q = 3000$ (M1)

$= 5.630120411$

$\therefore$ The number of complete years is 6. — (A1)

Exercise 2.5

(a) The gradient

$= \frac{1-(-9)}{-8-0}$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ (M1)

$= -\frac{5}{4}$ (A1)

(b) (i) $(-4, -4)$ (A1)

(ii) The exact distance

$= \sqrt{(-4-(-8))^2 + (-4-1)^2}$ $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (M1)

$= \sqrt{41}$ (A1)

(c) The required slope

$= -1 \div -\frac{5}{4}$ $\quad m_1 \times m_2 = -1$ (M1)

$= \frac{4}{5}$

The equation:

$y - 1 = \frac{4}{5}(x-(-8))$ $\quad y - y_1 = m(x - x_1)$ (M1)

$5(y-1) = 4(x+8)$

$5y - 5 = 4x + 32$

$4x - 5y + 37 = 0$ (A1)