Chapter 20 Solution

Quick Practice

Part I Solution

(a) (1) The area of the rectangle ABCD

$$= (\text{AD})(\text{CD})$$
$$= (e^{-1} - e^{-2})(1)$$
$$= \frac{e-1}{e^2}$$

(2) The area of the triangle ACD

$$= \frac{e-1}{e^2} \div 2$$
$$= \frac{e-1}{2e^2}$$

(3) $0 < A_1 < \frac{e-1}{2e^2}$

(b) (1) The area of the triangle CFG

$$= \frac{(\text{CG})(\text{FG})}{2}$$
$$= \frac{(e^{-2} - e^{-3})(1)}{2}$$
$$= \frac{e^{-2} - e^{-3}}{2}$$
$$= \frac{e-1}{2e^3}$$

(2) $0 < A_2 < \frac{e-1}{2e^3}$

(c) $0 < A_3 < \dfrac{(e^{-3} - e^{-4})(1)}{2}$

$0 < A_3 < \dfrac{e^{-3} - e^{-4}}{2}$

$0 < A_3 < \dfrac{e-1}{2e^4}$

(d) $0 < A_n < \dfrac{(e^{-n} - e^{-(n+1)})(1)}{2}$

$0 < A_n < \dfrac{e^{-n} - e^{-n-1}}{2}$

$0 < A_n < \dfrac{e-1}{2e^{n+1}}$

(e) (1) $A_n = \int_n^{n+1} e^{-x}\,\mathrm{d}x - (1)(e^{-(n+1)})$

$A_n = \left[-e^{-x}\right]_n^{n+1} - e^{-(n+1)}$

$A_n = -e^{-(n+1)} - (-e^{-n}) - e^{-(n+1)}$

$A_n = -2e^{-(n+1)} + e^{-n}$

$A_n = \dfrac{e-2}{e^{n+1}}$

(2) $A_{n+2} = \dfrac{e-2}{e^{(n+2)+1}}$

$A_{n+2} = \dfrac{e-2}{e^{n+3}}$

(3) $A_n - A_{n+2} = \dfrac{e-2}{e^{n+1}} - \dfrac{e-2}{e^{n+3}}$

$A_n - A_{n+2} = \dfrac{e^2(e-2) - (e-2)}{e^{n+3}}$

$A_n - A_{n+2} = \dfrac{(e^2 - 1)(e-2)}{e^{n+3}}$

$A_n - A_{n+2} = \dfrac{(e+1)(e-1)(e-2)}{e^{n+3}}$

(f) $e^{23}A_\beta - e + 2 = 0$

$e^{23}A_\beta = e - 2$

$A_\beta = \frac{e-2}{e^{23}}$

$\frac{e-2}{e^{\beta+1}} = \frac{e-2}{e^{23}}$

$\therefore \beta + 1 = 23$

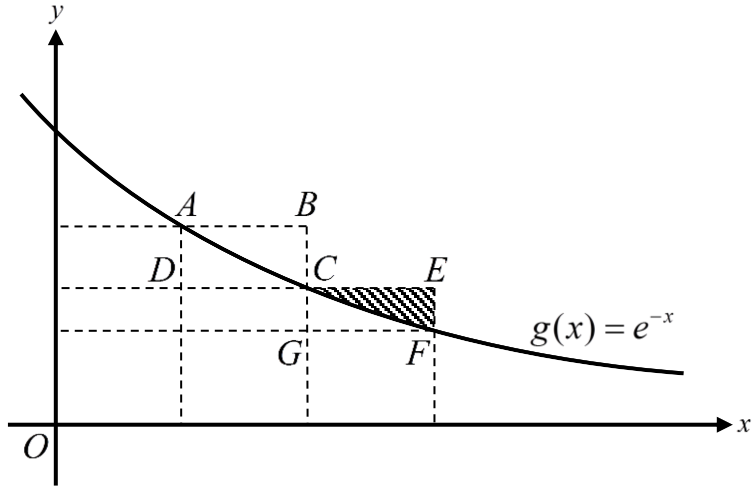
$\beta = 22$

Part II Solution

(g) (1) $\frac{e-1}{2e^2} < B_1 < 2\left(\frac{e-1}{2e^2}\right)$

$\frac{e-1}{2e^2} < B_1 < \frac{e-1}{e^2}$

(2)



(3) $\frac{e-1}{2e^3} < B_2 < \frac{e-1}{e^3}$, $\frac{e-1}{2e^4} < B_3 < \frac{e-1}{e^4}$

(4) $\frac{e-1}{2e^{n+1}} < B_n < \frac{e-1}{e^{n+1}}$

Part III Solution

(h) (1) $A_1 + B_1 = \frac{e-1}{e^2}$

$B_1 = \frac{e-1}{e^2} - A_1$

(2) $A_2 + B_2 = \frac{e-1}{e^3}$

$B_2 = \frac{e-1}{e^3} - A_2$

(3) $A_n + B_n = \frac{e-1}{e^{n+1}}$

$B_n = \frac{e-1}{e^{n+1}} - A_n$

(i) (1) Concave upward

(2) The area of A_n is always less than the area of B_n.

(j) (1) 0

(2) $A_n + B_n = \frac{e-1}{e^{n+1}}$

$0 < B_n < A_n < A_n + B_n$

$\therefore 0 < B_n < A_n < \frac{e-1}{e^{n+1}}$

$\lim_{n\to\infty} 0 = 0$

$\lim_{n\to\infty} \frac{e-1}{e^{n+1}} = \lim_{n\to\infty} \frac{e}{e^{n+1}} - \lim_{n\to\infty} \frac{1}{e^{n+1}}$

$\lim_{n\to\infty} \frac{e-1}{e^{n+1}} = \lim_{n\to\infty} \frac{1}{e^{n}} - \lim_{n+1\to\infty} \frac{1}{e^{n+1}}$

$\lim_{n\to\infty} \frac{e-1}{e^{n+1}} = 0 - 0$

$\lim_{n\to\infty} \frac{e-1}{e^{n+1}} = 0$

$\therefore \lim_{n\to\infty} A_n = \lim_{n\to\infty} B_n = 0$

Part IV Solution

(k) (1) $$\frac{e-1}{2e^2} < B_1 < \frac{e-1}{e^2}$$

$$\frac{e-1}{2e^3} < B_2 < \frac{e-1}{e^3}$$

$$\frac{e-1}{2e^4} < B_3 < \frac{e-1}{e^4}$$

$$\cdots$$

$$\frac{e-1}{2e^{n+1}} < B_n < \frac{e-1}{e^{n+1}}$$

$$\therefore \frac{e-1}{2e^2} + \frac{e-1}{2e^3} + \frac{e-1}{2e^4} + \cdots + \frac{e-1}{2e^{n+1}}$$

$$< B_1 + B_2 + B_3 + \cdots + B_n < \frac{e-1}{e^2} + \frac{e-1}{e^3} + \frac{e-1}{e^4} + \cdots + \frac{e-1}{e^{n+1}}$$

$$\frac{1}{2}\left(\frac{e-1}{e^2} + \frac{e-1}{e^3} + \frac{e-1}{e^4} + \cdots + \frac{e-1}{e^{n+1}}\right) < \sum_{k=1}^{n} B_k < \frac{e-1}{e^2} + \frac{e-1}{e^3} + \frac{e-1}{e^4} + \cdots + \frac{e-1}{e^{n+1}}$$

$$\frac{1}{2}\left(\frac{1}{e} - \frac{1}{e^2} + \frac{1}{e^2} - \frac{1}{e^3} + \frac{1}{e^3} - \frac{1}{e^4} + \cdots + \frac{1}{e^n} - \frac{1}{e^{n+1}}\right)$$

$$< \sum_{k=1}^{n} B_k < \frac{1}{e} - \frac{1}{e^2} + \frac{1}{e^2} - \frac{1}{e^3} + \frac{1}{e^3} - \frac{1}{e^4} + \cdots + \frac{1}{e^n} - \frac{1}{e^{n+1}}$$

$$\frac{1}{2}\left(\frac{1}{e} - \frac{1}{e^{n+1}}\right) < \sum_{k=1}^{n} B_k < \frac{1}{e} - \frac{1}{e^{n+1}}$$

$$\frac{1}{2}\left(\frac{e^n}{e^{n+1}} - \frac{1}{e^{n+1}}\right) < \sum_{k=1}^{n} B_k < \frac{e^n}{e^{n+1}} - \frac{1}{e^{n+1}}$$

$$\frac{e^n - 1}{2e^{n+1}} < \sum_{k=1}^{n} B_k < \frac{e^n - 1}{e^{n+1}}$$

(2) $$\lim_{n\to\infty} \frac{e^n - 1}{e^{n+1}} = \lim_{n\to\infty}\left(\frac{1}{e} - \frac{1}{e^{n+1}}\right)$$

$$\lim_{n\to\infty} \frac{e^n - 1}{e^{n+1}} = \frac{1}{e} - 0$$

$$\lim_{n\to\infty} \frac{e^n - 1}{e^{n+1}} = \frac{1}{e}$$

(3) $$\lim_{n\to\infty}\frac{e^n-1}{2e^{n+1}}<\lim_{n\to\infty}\sum_{k=1}^{n}B_k<\lim_{n\to\infty}\frac{e^n-1}{e^{n+1}}$$

$$\frac{1}{2}\lim_{n\to\infty}\frac{e^n-1}{e^{n+1}}<\lim_{n\to\infty}\sum_{k=1}^{n}B_k<\lim_{n\to\infty}\frac{e^n-1}{e^{n+1}}$$

$$\frac{1}{2}\left(\frac{1}{e}\right)<\lim_{n\to\infty}\sum_{k=1}^{n}B_k<\frac{1}{e}$$

$$\frac{1}{2e}<\lim_{n\to\infty}\sum_{k=1}^{n}B_k<\frac{1}{e}$$

(l) (1) $\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots$ is the sum to infinity of the geometric sequence with the first term $\frac{1}{e}$ and the common ratio $-\frac{1}{e}$.

$$\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots=\frac{\frac{1}{e}}{1-\left(-\frac{1}{e}\right)}$$

$$\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots=\frac{\frac{1}{e}}{1+\frac{1}{e}}$$

$$\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots=\frac{1}{e+1}$$

(2) $$\frac{e-1}{2e^2}<B_1<\frac{e-1}{e^2}$$

$$\frac{e-1}{2e^4}<B_3<\frac{e-1}{e^4}$$

$$\frac{e-1}{2e^6}<B_5<\frac{e-1}{e^6}$$

$\cdots$

$$\frac{e-1}{2e^{2n}}<B_{2n-1}<\frac{e-1}{e^{2n}}$$

$\cdots$

$$\therefore\frac{e-1}{2e^2}+\frac{e-1}{2e^4}+\frac{e-1}{2e^6}+\cdots+\frac{e-1}{2e^{2n}}+\cdots$$

$$<B_1+B_3+B_5+\cdots+B_{2n-1}+\cdots<\frac{e-1}{e^2}+\frac{e-1}{e^4}+\frac{e-1}{e^6}+\cdots+\frac{e-1}{e^{2n}}+\cdots$$

$$\frac{1}{2}\left(\frac{e-1}{e^2}+\frac{e-1}{e^4}+\frac{e-1}{e^6}+\cdots+\frac{e-1}{e^{2n}}+\cdots\right)$$
$$<B_1+B_3+B_5+\cdots+B_{2n-1}+\cdots<\frac{e-1}{e^2}+\frac{e-1}{e^4}+\frac{e-1}{e^6}+\cdots+\frac{e-1}{e^{2n}}+\cdots$$
$$\frac{1}{2}\left(\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots+\frac{1}{e^{2n-1}}-\frac{1}{e^{2n}}+\cdots\right)$$
$$<B_1+B_3+B_5+\cdots+B_{2n-1}+\cdots$$
$$<\frac{1}{e}-\frac{1}{e^2}+\frac{1}{e^3}-\frac{1}{e^4}+\frac{1}{e^5}-\frac{1}{e^6}+\cdots+\frac{1}{e^{2n-1}}-\frac{1}{e^{2n}}+\cdots$$
$$\frac{1}{2}\left(\frac{1}{e+1}\right)<B_1+B_3+B_5+\cdots+B_{2n-1}+\cdots<\frac{1}{e+1}$$
$$\frac{1}{2(e+1)}<B_1+B_3+B_5+\cdots+B_{2n-1}+\cdots<\frac{1}{e+1}$$

(m) (1)

$$B_k=(1)(e^{-k})-\int_k^{k+1}e^{-x}\mathrm{d}x$$
$$B_k=e^{-k}-\left[-e^{-x}\right]_k^{k+1}$$
$$B_k=e^{-k}-(-e^{-(k+1)}-(-e^{-k}))$$
$$B_k=e^{-k}+e^{-k-1}-e^{-k}$$
$$B_k=e^{-k-1}$$

(2)

$$B_k=e^{-k-1}$$
$$\frac{\mathrm{d}B_k}{\mathrm{d}k}=(e^{-k-1})(-1)$$
$$\frac{\mathrm{d}B_k}{\mathrm{d}k}=-e^{-k-1}$$
$$\frac{\mathrm{d}B_k}{\mathrm{d}k}<0 \text{ for } k\geq 1$$

Thus, B_k is decreasing.

Exercise 81

1. (a) (1) $I(0)=\int_0^1 \frac{1}{\sqrt{1-x^2}}\mathrm{d}x$

$I(0)=\left[\arcsin x\right]_0^1$ A1

$I(0)=\arcsin 1-\arcsin 0$

$I(0)=\frac{\pi}{2}-0$

$I(0)=\frac{\pi}{2}$ A1

(2) $I(1)=\int_0^1 \frac{x}{\sqrt{1-x^2}}\mathrm{d}x$

Let $u=1-x^2$. M1

$\frac{\mathrm{d}u}{\mathrm{d}x}=-2x \Rightarrow -\frac{1}{2}\mathrm{d}u=x\mathrm{d}x$

$x=1 \Rightarrow u=1-1^2=0$

$x=0 \Rightarrow u=1-0^2=1$

$\therefore I(1)=\int_1^0 -\frac{1}{2}\cdot\frac{1}{\sqrt{u}}\mathrm{d}u$

$I(1)=-\frac{1}{2}\left[2\sqrt{u}\right]_1^0$ A1

$I(1)=-\frac{1}{2}(0-2)$

$I(1)=1$ A1

(3) $I(n)=\int_0^1 \frac{x^n}{\sqrt{1-x^2}}\mathrm{d}x$

Let $\theta=\sqrt{1-x^2}$. M1

$\frac{\mathrm{d}\theta}{\mathrm{d}x}=\frac{1}{2\sqrt{1-x^2}}(-2x)=-\frac{x}{\sqrt{1-x^2}}$

$\Rightarrow -\frac{1}{x}\mathrm{d}\theta=\frac{1}{\sqrt{1-x^2}}\mathrm{d}x$

$\therefore I(n)=\int_0^1 -\frac{x^n}{x}\mathrm{d}(\sqrt{1-x^2})$ A1

$I(n)=\int_0^1 -x^{n-1}\mathrm{d}(\sqrt{1-x^2})$

$I(n)=\left[-x^{n-1}\sqrt{1-x^2}\right]_0^1-\int_0^1\sqrt{1-x^2}\mathrm{d}(-x^{n-1})$ A1

$I(n)=(-1^{n-1}\sqrt{1-1^2})-(-0^{n-1}\sqrt{1-0^2})$

$-\int_0^1\sqrt{1-x^2}\mathrm{d}(-x^{n-1})$

$I(n)=-\int_0^1\sqrt{1-x^2}\cdot-(n-1)x^{n-2}\mathrm{d}x$

$I(n)=(n-1)\int_0^1\sqrt{1-x^2}\cdot x^{n-2}\mathrm{d}x$ A1

$I(n)=(n-1)\int_0^1\frac{(1-x^2)x^{n-2}}{\sqrt{1-x^2}}\mathrm{d}x$ M1

$I(n)=(n-1)\int_0^1\frac{x^{n-2}-x^n}{\sqrt{1-x^2}}\mathrm{d}x$

$I(n)=(n-1)\left(\int_0^1\frac{x^{n-2}}{\sqrt{1-x^2}}\mathrm{d}x-\int_0^1\frac{x^n}{\sqrt{1-x^2}}\mathrm{d}x\right)$

$I(n)=(n-1)(I(n-2)-I(n))$ A1

$I(n)=(n-1)I(n-2)-(n-1)I(n)$

$nI(n)=(n-1)I(n-2)$

$I(n)=\frac{n-1}{n}I(n-2)$ AG

(4) $I(3)=\frac{3-1}{3}I(1)$

$I(3)=\frac{2}{3}(1)$

$I(3)=\frac{2}{3}$ A1

[12]

(b) (1) $H(0)=\int_0^1 \frac{1}{(x^2+1)\sqrt{1-x^2}}\,dx$

Let $x=\tan\theta$. M1

$\frac{dx}{d\theta}=\sec^2\theta \Rightarrow dx=\sec^2\theta d\theta$

$x=1\Rightarrow \tan\theta=1\Rightarrow \theta=\frac{\pi}{4}$

$x=0\Rightarrow \tan\theta=0\Rightarrow \theta=0$

$\therefore H(0)=\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{(\tan^2\theta+1)\sqrt{1-\tan^2\theta}}\,d\theta$ A1

$H(0)=\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{\sec^2\theta\sqrt{1-\tan^2\theta}}\,d\theta$ A1

$H(0)=\int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{1-\tan^2\theta}}\,d\theta$

$H(0)=\int_0^{\frac{\pi}{4}} \frac{\cos\theta}{\sqrt{\cos^2\theta-\sin^2\theta}}\,d\theta$ M1

$H(0)=\int_0^{\frac{\pi}{4}} \frac{\cos\theta}{\sqrt{1-\sin^2\theta-\sin^2\theta}}\,d\theta$ A1

$H(0)=\int_0^{\frac{\pi}{4}} \frac{\cos\theta}{\sqrt{1-2\sin^2\theta}}\,d\theta$ AG

(2) $H(0)=\int_0^{\frac{\pi}{4}} \frac{\cos\theta}{\sqrt{1-2\sin^2\theta}}\,d\theta$

Let $v=\sqrt{2}\sin\theta$. M1

$\frac{dv}{d\theta}=\sqrt{2}\cos\theta \Rightarrow \frac{1}{\sqrt{2}}dv=\cos\theta d\theta$

$\theta=\frac{\pi}{4}\Rightarrow v=\sqrt{2}\sin\frac{\pi}{4}=1$

$\theta=0\Rightarrow v=\sqrt{2}\sin 0=0$

$\therefore H(0)=\int_0^1 \frac{1}{\sqrt{2}\cdot\sqrt{1-v^2}}\,dv$ A1

$H(0)=\frac{1}{\sqrt{2}}\int_0^1 \frac{1}{\sqrt{1-v^2}}\,dv$

$H(0)=\frac{\sqrt{2}}{2}I(0)$ AG

[7]

(c) (1) $H(2)=\int_0^1 \frac{x^2}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(2)=\int_0^1 \frac{x^2+1-1}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$ M1

$H(2)=\int_0^1 \frac{x^2+1}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$-\int_0^1 \frac{1}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(2)=\int_0^1 \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x-\int_0^1 \frac{1}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(2)=I(0)-H(0)$ A1

(2) $H(3)=\int_0^1 \frac{x^3}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(3)=\int_0^1 \frac{x^3+x-x}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$ M1

$H(3)=\int_0^1 \frac{x(x^2+1)}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$-\int_0^1 \frac{x}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(3)=\int_0^1 \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x-\int_0^1 \frac{x}{(x^2+1)\sqrt{1-x^2}}\,\mathrm{d}x$

$H(3)=I(1)-H(1)$ A1

(3) $H(n)=I(n-2)-H(n-2)$ A1

[5]

(d) $H(4) = I(2) - H(2)$ M1

$$H(4) = I(2) - (I(0) - H(0))$$

$$H(4) = \frac{2-1}{2} I(0) - I(0) + H(0)$$ M1

$$H(4) = -\frac{1}{2} I(0) + H(0)$$

$$H(4) = -\frac{1}{2} I(0) + \frac{\sqrt{2}}{2} I(0)$$ M1

$$H(4) = -\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\pi}{2}\right)$$

$$H(4) = \frac{\pi(\sqrt{2}-1)}{4}$$ A1

[4]

2. (a) $z^n = 1$

$z^n = \cos 0 + \mathrm{i}\sin 0$ A1

$z = \cos\left(\dfrac{0+2k\pi}{n}\right) + \mathrm{i}\sin\left(\dfrac{0+2k\pi}{n}\right)$ M1

$(k = 0, 1, 2, \cdots, n-1)$

$z = \cos\dfrac{2k\pi}{n} + \mathrm{i}\sin\dfrac{2k\pi}{n}$

Thus, $\arg(\omega_k) = \dfrac{2k\pi}{n}$. AG

[2]

(b) (1) $\sin \mathrm{O\hat{P}_0P_1} = \dfrac{d_3}{\mathrm{OP_0}}$ M1

$\sin\left(\dfrac{\pi}{2} - \dfrac{\frac{2\pi}{3}}{2}\right) = \dfrac{d_3}{1}$ A1

$\cos\dfrac{\pi}{3} = \dfrac{d_3}{1}$

$d_3 = \dfrac{1}{2}$ A1

(2) $\sin \mathrm{O\hat{P}_0P_1} = \dfrac{d_4}{\mathrm{OP_0}}$ M1

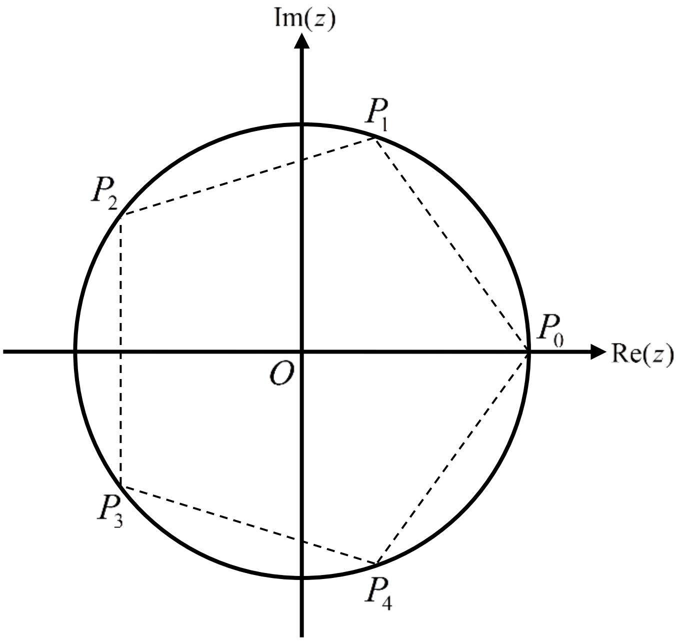
$\sin\left(\dfrac{\pi}{2} - \dfrac{\frac{2\pi}{4}}{2}\right) = \dfrac{d_4}{1}$ A1

$\cos\dfrac{\pi}{4} = \dfrac{d_4}{1}$

$d_4 = \dfrac{\sqrt{2}}{2}$ AG

(3) A1 for regular pentagon

A1 for correct arguments for P_i



(4) $d_5 = \cos\frac{\pi}{5}$ A1

(5) $\sin \mathrm{O\hat{P}_0P_1} = \frac{d_n}{\mathrm{OP_0}}$ M1

$\sin\left(\frac{\pi}{2} - \frac{\frac{2\pi}{n}}{2}\right) = \frac{d_n}{1}$ A1

$\cos\frac{\pi}{n} = \frac{d_n}{1}$

$d_n = \cos\frac{\pi}{n}$ AG

[10]

(c) (1) $A_3 = 3\left(\frac{1}{2}(\mathrm{OP_0})(\mathrm{OP_1})\sin \mathrm{P_0\hat{O}P_1}\right)$ M1

$A_3 = 3\left(\frac{1}{2}(1)(1)\sin\frac{2\pi}{3}\right)$ A1

$A_3 = 3\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$A_3 = \frac{3\sqrt{3}}{4}$ AG

(2) $A_4 = 4\left(\frac{1}{2}(\mathrm{OP_0})(\mathrm{OP_1})\sin \mathrm{P_0\hat{O}P_1}\right)$ M1

$A_4 = 4\left(\frac{1}{2}(1)(1)\sin\frac{2\pi}{4}\right)$ A1

$A_4 = 4\left(\frac{1}{2}\right)(1)$

$A_4 = 2$ A1

(3) $A_n = n\left(\frac{1}{2}(\mathrm{OP_0})(\mathrm{OP_1})\sin \mathrm{P_0\hat{O}P_1}\right)$ M1

$A_n = n\left(\frac{1}{2}(1)(1)\sin\frac{2\pi}{n}\right)$ A1

$A_n = \frac{n}{2}\sin\frac{2\pi}{n}$ AG

[7]

(d) $A_n = \frac{n}{2}\sin\frac{2\pi}{n}$

$A_n = \frac{n}{2}\left(2\sin\frac{\pi}{n}\cos\frac{\pi}{n}\right)$ A1

$A_n = n\left(\sqrt{1-\cos^2\frac{\pi}{n}}\right)\cos\frac{\pi}{n}$

$A_n = nd_n(\sqrt{1-d_n{}^2})$ A1

[2]

(e) (1) $$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \lim_{n\to\infty} n\sqrt{1-\cos^2\frac{\pi}{n}}$$

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \lim_{n\to\infty} n\sin\frac{\pi}{n}$$ A1

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \lim_{n\to\infty} \frac{\sin\frac{\pi}{n}}{\frac{1}{n}}$$

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \lim_{n\to\infty} \frac{\left(\cos\frac{\pi}{n}\right)\left(-\frac{\pi}{n^2}\right)}{-\frac{1}{n^2}} \left(\because \frac{0}{0}\right)$$ M1A1

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \lim_{n\to\infty} \pi\cos\frac{\pi}{n}$$

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \pi\cos 0$$

$$\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}} = \pi$$ AG

(2) $$\lim_{n\to\infty} A_n = \lim_{n\to\infty} nd_n(\sqrt{1-d_n^{\ 2}})$$

$$\lim_{n\to\infty} A_n = \left(\lim_{n\to\infty} d_n\right)\left(\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}}\right)$$

$$\lim_{n\to\infty} A_n = \left(\lim_{n\to\infty} \cos\frac{\pi}{n}\right)\left(\lim_{n\to\infty} n\sqrt{1-d_n^{\ 2}}\right)$$ M1

$$\lim_{n\to\infty} A_n = (\cos 0)(\pi)$$

$$\lim_{n\to\infty} A_n = \pi$$ A1

(3) The inscribed polygon becomes a unit circle and the shortest distance of the boundary of the polygon to the origin becomes the radius 1 as n tends to positive infinity. R1

[6]