AI HL Practice Set 2 Paper 2 Solution

1. (a) $7(98)+24f-2990=0$ (M1) for setting equation

$24f=2304$

$f=96$ A1

[2]

(b) $-\frac{7}{24}$ A1

[1]

(c) (i) The gradient of DE

$=-1\div -\frac{7}{24}$ (M1) for valid approach

$=\frac{24}{7}$ A1

(ii) The equation of DE:

$y-10=\frac{24}{7}(x-125)$ M1A1

$7y-70=24(x-125)$ A1

$7y-70=24x-3000$

$24x-7y-2930=0$ AG

[5]

(d) $(146, 82)$ A2

[2]

(e) The coordinates of the mid-point of CD

$=\left(\frac{50+146}{2}, \frac{110+82}{2}\right)$ M1A1

$=(98, 96)$

Thus, F is the mid-point of CD. AG

[2]

(f) The length of DE

$=\sqrt{(146-125)^2+(82-10)^2}$ (A1) for substitution

$=75$ A1

[2]

(g) The area of the triangle CDE

$= \frac{(75)(100)}{2}$ (M1) for valid approach

$= 3750 \text{ m}^2$ A1

[2]

(h) The total area

$= 3750 + \frac{(BC + AE)(AB)}{2}$ (M1)(A1) for correct approach

$= 3750 + \frac{(40 + 115)(100)}{2}$ (A1) for substitution

$= 11500 \text{ m}^2$ A1

[4]

2. (a) $H_1: \mu_1 > \mu_2$ A1

[1]

(b) $p\text{-value} = 0.0231895114$ (A1) for correct value

$p\text{-value} = 0.0232$ A1

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1

[2]

(d) (i) The required probability

$=\left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$=\frac{1}{9}$ A1

(ii) The required probability

$=\left(\frac{5}{10}\right)\left(\frac{2}{9}\right)+\left(\frac{5}{10}\right)\left(\frac{7}{9}\right)+\left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$=\frac{11}{18}$ A1

[4]

(e) H_1: The age and the reading preference are not independent. A1

[1]

(f) 4 A1

[1]

(g) $\chi^2_{calc} = 53.64204545$ (A1) for correct value

$\chi^2_{calc} = 53.6$ A1

[2]

(h) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 13.277$. R1

[2]

3.	(a)	$f'(x) = -3x^2 + b(2x) - 432(1) + 0$	(A1) for correct derivatives
		$f'(x) = -3x^2 + 2bx - 432$	
		$f'(8) = 0$	(M1) for setting equation
		$\therefore -3(8)^2 + 2b(8) - 432 = 0$	(A1) for substitution
		$16b = 624$	
		$b = 39$	A1
			[4]
	(b)	(i) 984	A1
		(ii) $(18, 1484)$	A2
			[3]
	(c)	$8 < x < 18$	A2
			[2]
	(d)	(i) $984 < k < 1484$	A2
		(ii) $k \le 984$ or $k \ge 1484$	A2
			[4]
	(e)	$C(x) = -x^3 + 39x^2 - 432x + 2456$	
		$C(8) = 984$	
		$C(25)$	
		$= -25^3 + 39(25)^2 - 432(25) + 2456$	A1
		$= 406$	
		$C(8) > C(25)$	R1
		Thus, the average cost attains its minimum when 25000 smart watches are produced.	AG
			[2]
	(f)	$C(x) \le 984$	(M1) for setting inequality
		$-x^3 + 39x^2 - 432x + 2456 \le 984$	
		$-x^3 + 39x^2 - 432x + 1472 \le 0$	
		By considering the graph of $y = -x^3 + 39x^2 - 432x + 1472$, $x = 8$ or $x \ge 23$.	
		Thus, the range of values of x are $x = 8$ or $23 \le x \le 25$.	A2
			[3]

4. (a) The initial velocity

$= v(0)$

$= -0.5(0-5)^3$ — (M1) for substitution

$= 62.5 \text{ ms}^{-1}$ — A1

[2]

(b) $v(t) = -13.5$ — (M1) for setting equation

$-0.5(t-5)^3 = -13.5$

$(t-5)^3 = 27$

$t-5 = 3$ — (A1) for correct approach

$t = 8$ — A1

[3]

(c) The total distance travelled

$= \int_0^{10} |v(t)| \, \mathrm{d}t$ — (M1) for valid approach

$= \int_0^{10} \left|-0.5(t-5)^3\right| \mathrm{d}t$ — (A1) for substitution

$= 156.25 \text{ m}$ — A1

[3]

(d) $a(t) = v'(t)$

$a(t) = -0.5(3)(t-5)^2(1)$ — (A1) for correct approach

$a(t) = -1.5(t-5)^2$ — A1

[2]

(e) $v(t) \geq 0$ and $a(t) \geq 0$

By considering the graph of $y = -0.5(t-5)^3$ and

$y = -1.5(t-5)^2$, $0 \leq t \leq 5$ and $t = 5$. — R2

$\therefore t = 5$ — A1

[3]

(f) $s(t) = \int v(t) \mathrm{d}t$

$s(t) = \int -0.5(t-5)^3 \mathrm{d}t$ — (A1) for correct approach

$s(t) = -0.125(t-5)^4 + C$ — A1

$-78 = -0.125(0-5)^4 + C$ — (M1) for substitution

$C = 0.125$

$\therefore s(t) = -0.125(t-5)^4 + 0.125$ — A1

[4]

5. (a) $L_1:\begin{cases} x=3+2t \\ y=6-6t \\ z=9-2t \end{cases}$, $L_2:\begin{cases} x=1+3s \\ y=-2-2s \\ z=3+s \end{cases}$ M1

$9-2t=3+s$

$s=6-2t$

$3+2t=1+3s$

$\therefore 3+2t=1+3(6-2t)$ M1

$3+2t=19-6t$

$8t=16$

$t=2$

$\therefore s=6-2(2)=2$ A1

$\begin{cases} x=3+2(2)=7 \\ y=6-6(2)=-6 \\ z=9-2(2)=5 \end{cases}$ M1

Thus, the coordinates of C are $(7,-6,5)$. AG

[4]

(b) $(3\mathbf{i}-2\mathbf{j}+\mathbf{k})\cdot\mathbf{k}=|3\mathbf{i}-2\mathbf{j}+\mathbf{k}||\mathbf{k}|\cos\theta$ (M1) for valid approach

$(3)(0)+(-2)(0)+(1)(1)=(\sqrt{3^2+(-2)^2+1^2})(1)\cos\theta$ (A1) for correct approach

$1=\sqrt{14}\cos\theta$

$\cos\theta=\frac{1}{\sqrt{14}}$

$\theta=1.300246564\text{ rad}$

$\theta=1.30\text{ rad}$ A1

[3]

(c) (i) $\overrightarrow{CA} = 6\mathbf{i} - 18\mathbf{j} - 6\mathbf{k}$ A1

(ii) $\overrightarrow{CB} = 9\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ A1

(iii) The required area

$= \frac{1}{2}\left|\overrightarrow{CA} \times \overrightarrow{CB}\right|$ (M1) for valid approach

$= \frac{1}{2}\left|\begin{pmatrix} (-18)(3) - (-6)(-6) \\ (-6)(9) - (6)(3) \\ (6)(-6) - (-18)(9) \end{pmatrix}\right|$ (A1) for substitution

$= \frac{1}{2}\left|-90\mathbf{i} - 72\mathbf{j} + 126\mathbf{k}\right|$

$= \frac{1}{2}\sqrt{(-90)^2 + (-72)^2 + 126^2}$

$= 85.38149682$

$= 85.4$ A1

[5]

(d) 171 A1

[1]

6. (a) Eulerian circuit does not exist. A1
As not all vertices are of even degree. R1

[2]

(b) BC A1

[1]

(c) For any three edges correct A1
For all edges correct A1

1. Choose BC of weight 6
2. Choose BG of weight 10
3. Choose GE of weight 11
4. Choose EF of weight 9
5. Choose AB of weight 17
6. Choose ED of weight 19

Thus, the minimum spanning tree is a tree containing BC, BG, GE, EF, AB and ED. A1

[3]

(d) 72 A1

[1]

(e) For any three edges correct A1
For all edges correct A1

1. Choose GB of weight 10
2. Choose BC of weight 6
3. Choose CD of weight 21
4. Choose DE of weight 19
5. Choose EF of weight 9
6. Choose FA of weight 18
7. Choose AG of weight 26

Thus, the required upper bound is 109. A1

[3]

(f) For any two edges correct A1
For all edges correct A1

1. Choose BC of weight 6
2. Choose EF of weight 9
3. Choose AB of weight 17
4. Choose AF of weight 18
5. Choose DE of weight 19

Therefore, the weight of a minimum spanning tree after deleting the vertex G is 69. A1
The required lower bound
$= 69 + 10 + 11$
$= 90$ A1

[4]

7. (a) The characteristic polynomial of $\mathbf{M}$

$$= \det(\mathbf{M} - \lambda\mathbf{I})$$

$$= \begin{vmatrix} \frac{5}{3} - \lambda & \frac{4}{3} \\ -\frac{2}{3} & -\frac{1}{3} - \lambda \end{vmatrix}$$ (M1) for valid approach

$$= \left(\frac{5}{3} - \lambda\right)\left(-\frac{1}{3} - \lambda\right) - \left(\frac{4}{3}\right)\left(-\frac{2}{3}\right)$$

$$= -\frac{5}{9} - \frac{4}{3}\lambda + \lambda^2 + \frac{8}{9}$$

$$= \lambda^2 - \frac{4}{3}\lambda + \frac{1}{3}$$ A1

[2]

(b) $\lambda_1 = \frac{1}{3}$, $\lambda_2 = 1$ A2

[2]

(c) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix} 1 & 1 \\ -1 & -\frac{1}{2} \end{pmatrix}$ A1

(ii) $\begin{pmatrix} \left(\frac{1}{3}\right)^n & 0 \\ 0 & 1 \end{pmatrix}$ A2

[3]

(e) $\mathbf{M}^n$

$$=\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}\left(\frac{1}{3}\right)^n & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}^{-1}$$

$$=\begin{pmatrix}\left(\frac{1}{3}\right)^n & 1\\ -\left(\frac{1}{3}\right)^n & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & -\frac{1}{2}\end{pmatrix}^{-1}$$ A1

$$=\begin{pmatrix}\left(\frac{1}{3}\right)^n & 1\\ -\left(\frac{1}{3}\right)^n & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}-1 & -2\\ 2 & 2\end{pmatrix}$$ (A1) for correct approach

$$=\begin{pmatrix}-\left(\frac{1}{3}\right)^n+2 & -2\left(\frac{1}{3}\right)^n+2\\ \left(\frac{1}{3}\right)^n-1 & 2\left(\frac{1}{3}\right)^n-1\end{pmatrix}$$ A1

[3]

(f) $\lim_{n\to\infty} g(n)=2$ A1

[1]