Exercise 5.1

(a) $f'(x)$

$= 2(2x) - 1 - 0 - 4(-2x^{-3})$ — $2x$, 1, 0 & $-2x^{-3}$ (A1)

$= 4x - 1 + 8x^{-3}$ — (A1)

(b) The gradient of L

$= 4(2) - 1 + 8(2)^{-3}$ — $x = 2$ (M1)

$= 8$ — (A1)

(c) The equation of L:

$y - 4 = 8(x - 2)$ — $y - y_1 = m(x - x_1)$ (M1)

$y - 4 = 8x - 16$

$y = 8x - 12$ — (A1)

(d) (i) $-\frac{1}{8}$ — (A1)

(ii) The equation of the normal:

$y - 4 = -\frac{1}{8}(x - 2)$ — $y - y_1 = m(x - x_1)$ (M1)

$y - 4 = -\frac{1}{8}x + \frac{1}{4}$

$8y - 32 = -x + 2$

$x + 8y - 34 = 0$ — (A1)

Exercise 5.2

(a) $C'(x)$

$= 2x + 0 + 54(-1x^{-2})$ — $2x$, 0 & $-1x^{-2}$ (A1)

$= 2x - \frac{54}{x^2}$ — (A1)

(b) By considering the graph of $y = x^2 + 8 + \frac{54}{x}$, the coordinates of the minimum point are $(3, 35)$. — GDC approach (M1)

Thus, the required mass of a titanium block is 3 kg. — (A1)

(c) $\$35$ — (A1)

Exercise 5.3

(a) (i) $336 = \left(\frac{1}{4}\pi r^2\right)(h)$ — $V = \frac{1}{4}\pi r^2 h$ (A1)

$h = \frac{1344}{\pi r^2}$ — (A1)

(ii) A

$= \left(\frac{1}{4}\pi r^2\right)(2) + (rh)(2) + \frac{1}{4}(2\pi r)(h)$ — Sum of five areas (A1)

$= \frac{1}{2}\pi r^2 + 2rh + \frac{1}{2}\pi rh$

$= \frac{1}{2}\pi r^2 + 2r\left(\frac{1344}{\pi r^2}\right) + \frac{1}{2}\pi r\left(\frac{1344}{\pi r^2}\right)$ — $h = \frac{1344}{\pi r^2}$ (M1)

$= \frac{1}{2}\pi r^2 + \frac{2688}{\pi r} + \frac{672}{r}$ — $\frac{1}{2}\pi r^2 + \frac{2688}{\pi r} + \frac{672}{r}$ (A1)

$\therefore A = \frac{1}{2}\pi r^2 + \frac{1}{r}\left(\frac{2688}{\pi} + 672\right)$ — (AG)

(b) $\frac{\mathrm{d}A}{\mathrm{d}r}$

$= \frac{1}{2}\pi(2r) + \left(\frac{2688}{\pi} + 672\right)(-1r^{-2})$ — $2r$ & $-1r^{-2}$ (A1)

$= \pi r - \left(\frac{2688}{\pi} + 672\right)\frac{1}{r^2}$ — (A1)

(c) $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ — $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ (M1)

$\pi r - \left(\frac{2688}{\pi} + 672\right)\frac{1}{r^2} = 0$

By considering the graph of $y = \pi r - \left(\frac{2688}{\pi} + 672\right)\frac{1}{r^2}$, The horizontal intercept is $r = 7.8635996$. — GDC approach (M1)

Thus, $r = 7.86$. — (A1)

(d) The minimum total surface area

$=\frac{1}{2}\pi(7.8635996)^2+\frac{1}{7.8635996}\left(\frac{2688}{\pi}+672\right)$ $r=7.8635996$ (M1)

$=291.3964122$ cm^2

$=291$ cm^2 (A1)

(e) The minimum number of painting buckets

$=\frac{291.3964122}{70}$ $\frac{291.3964122}{70}$ (M1)

$=4.162805888$

Thus, 5 buckets are needed. (A1)

Exercise 5.4

(a) The actual area of R

$= \int_0^1 f(x)\mathrm{d}x$ — Definite integral (M1)

$= \int_0^1 (3\sqrt{x}+1)\mathrm{d}x$

$= 3.000000223$

$= 3.00$ — (A1)

(b) The percentage error

$= \left|\dfrac{\pi - 3.000000223}{3.000000223}\right| \times 100\%$ — $\left|\dfrac{v_A - v_E}{v_E}\right| \times 100\%$ (M1)

$= 4.719747335\%$

$= 4.72\%$ — (A1)

Exercise 5.5

(a) $f(x)$

$= \int \left(2x - \frac{1}{x^2}\right) dx$ — Indefinite integral (M1)

$= 2\left(\frac{1}{2}x^2\right) - \left(\frac{1}{-1}x^{-1}\right) + C$ — $\frac{1}{2}x^2$ & $\frac{1}{-1}x^{-1}$ (A1)

$= x^2 + \frac{1}{x} + C$

$7 = 2^2 + \frac{1}{2} + C$ — $f(2) = 7$ (M1)

$C = 2.5$ — 2.5 (A1)

$\therefore f(x) = x^2 + \frac{1}{x} + 2.5$ — (AG)

(b) The required area

$= \int_1^3 \left(x^2 + \frac{1}{x} + 2.5\right) dx$ — $\int_1^3 \left(x^2 + \frac{1}{x} + 2.5\right) dx$ (A1)

$= 14.76527896$

$= 14.8$ — (A1)

(c) (i) The required estimate

$= \frac{1}{2}\left(\frac{3-1}{4}\right)\left[\begin{array}{l} f(1) + f(3) \\ +2\left(f(1.5) + f(2) + f(2.5)\right) \end{array}\right]$ — $\frac{1}{2}h\left[\begin{array}{l} f(a) + f(b) \\ +2(f(x_1) + f(x_2) \\ +\ldots + f(x_{n-1})) \end{array}\right]$ (M1)

$= \frac{1}{2}\left(\frac{3-1}{4}\right)\left[4.5 + \frac{71}{6} + 2\left(\frac{65}{12} + 7 + 9.15\right)\right]$ — $f(a), f(x_1), \cdots, f(b)$ (A1)

$= 14.86666667$

$= 14.9$ — (A1)

(ii) Overestimates — (A1)