Exercise 1.1

(a) The required hypotenuse

$= \sqrt{1107^2 + 4920^2}$ — Pythagoras' theorem (A1)

$= 5043 \text{ mm}$

$= 5.043 \times 10^3 \text{ mm}$ — $a = 5.043$ & $k = 3$ (A1)

(b) The required perimeter

$= 1107 + 4920 + 5043$ — The sum of 3 sides (A1)

$= 11070 \text{ mm}$

$= 11000 \text{ mm}$ — Round off to 2 sig. fig.

$= 1.1 \times 10^4 \text{ mm}$ — $a = 1.1$ & $k = 4$ (A1)

(c) The required area

$= \dfrac{(1107)(4920)}{2}$ — $\dfrac{\text{Base length} \times \text{Height}}{2}$ (A1)

$= 2723220 \text{ mm}^2$

$= 2723000 \text{ mm}^2$ — Round off to 4 sig. fig.

$= 2.723 \times 10^6 \text{ mm}^2$ — $a = 2.723$ & $k = 6$ (A1)

Exercise 1.2

(a) The exact volume

$= (2)(0.8)(0.8)$ — $V = lwh$ (M1)

$= 1.28 \text{ m}^3$ — (A1)

(b) The approximated value

$= (2.2)(1)(0.7)$

$= 1.54 \text{ m}^3$ — 1.54 (A1)

The percentage error

$= \left| \dfrac{1.54 - 1.28}{1.28} \right| \times 100\%$ — $\left| \dfrac{v_A - v_E}{v_E} \right| \times 100\%$ (M1)

$= 20.3125\%$

$= 20.3\%$ — (A1)

Exercise 1.3

(a) $u_1 = 10$ (A1)

(b) (i) $u_2 = (5+1)(2)$

$u_2 = 12$

$u_3 = (5+1+1)(2)$ $u_2 = (6)(2)$ & $u_3 = (7)(2)$ (A1)

$u_3 = 14$ (AG)

(ii) $d = 2$ (A1)

(c) (i) $u_n = (20)(2)$ Set up an equation

$\therefore 10 + (n-1)(2) = 40$ Correct equation (A1)

$2(n-1) = 30$

$n - 1 = 15$

$n = 16$

Thus, Fatima has collected 16 apples. (A1)

(ii) The total distance

$= S_{16}$

$= \frac{16}{2}[u_1 + u_{16}]$ $S_n = \frac{n}{2}[u_1 + u_n]$ (M1)

$= \frac{16}{2}(10 + 40)$ $u_1 = 10$ & $u_{16} = 40$ (A1)

$= 400$ metres (A1)

(d) (i) $S_n = 491$

$\therefore \frac{n}{2}[2(10)+(n-1)(2)] = 491$ — Correct equation (A1)

$\frac{n}{2}(20+2n-2) = 491$

$\frac{n(2n+18)}{2} - 491 = 0$

By considering the graph of $y = \frac{n(2n+18)}{2} - 491$, the horizontal intercepts are -27.11084 (*Rejected*) and 18.110838. — GDC approach (M1)

Thus, the total number of apples that Akash has collected is 18. — (A1)

(ii) The required distance

$= 491 - S_{18}$ — Subtracted by S_{18} (M1)

$= 491 - \frac{18}{2}[2(10)+(18-1)(2)]$ — $u_1 = 10$ & $d = 2$ (A1)

$= 5$ metres — (A1)

Exercise 1.4

(a) The café's profit

$= u_{10}$ — 10th term (M1)

$= u_1 \times r^{10-1}$

$= 1200 \times 1.08^9$ — $u_1 = 1200$ & $r = 1.08$ (A1)

$= \$2398.805553$

$= \$2400$ — (A1)

(b) (i) S_n

$= \dfrac{u_1(1-r^n)}{1-r}$ — $S_n = \dfrac{u_1(1-r^n)}{1-r}$ (M1)

$= \dfrac{1200(1-1.08^n)}{1-1.08}$ — $u_1 = 1200$ & $r = 1.08$ (A1)

$= -15000(1-1.08^n)$ — (AG)

(ii) The total profit

$= S_{11}$

$= -15000(1-1.08^{11})$ — $n = 11$ (M1)

$= \$19974.58496$

$= \$20000$ — (A1)

(c) The dessert shop's profit

$= v_7$ — 7th term (M1)

$= 1200 + (7-1)(180)$ — $v_1 = 1200$ & $d = 180$ (A1)

$= \$2280$ — (A1)

(d) $S_m > \dfrac{m}{2}\left[2u_1 + (m-1)d\right]$ — Set up an inequality

$\therefore -15000(1-1.08^m) > \dfrac{m}{2}\left[2(1200) + (m-1)(180)\right]$ — Correct inequality (A1)

$-15000(1-1.08^m) - \dfrac{m}{2}\left[2(1200) + (m-1)(180)\right] > 0$

By considering the graph of

$y = -15000(1-1.08^m) - \dfrac{m}{2}\left[2(1200) + (m-1)(180)\right]$,

the graph is above the horizontal axis when

$m < 1$ (*Rejected*) or $m > 23.044309$. — GDC approach (M1)

$\therefore m = 24$ — (A1)

(e) The upper limit of the total profit

$= S_{\infty}$

$= \dfrac{u_1}{1-r}$ $S_{\infty} = \dfrac{u_1}{1-r}$ (M1)

$= \dfrac{1200}{1-0.95}$ $u_1 = 1200$ & $r = 0.95$ (A1)

$= \$24000$

$> \$22000$ $\$24000 > \$22000$ (R1)

Thus, the owner's claim is correct. (AG)

Exercise 1.5

(a) (i) $\begin{cases} 8x+7y+5z=41 \\ 6x+4y+10z=22 \\ 13x+7y=66 \end{cases}$ (A1)(A1)(A1)

(ii) $x=4$, $y=2$ and $z=-1$ (A1)(A1)(A1)

(b) A team drops one point for losing a game. (A1)

Exercise 1.6

(a) (i) By financial solver:

N(n) = 16
I% = 8
PV = −10000
PMT(Pmt) = 0
FV = ?
P / Y(PpY) = 4
C / Y(CpY) = 4
PMT(PmtAt) : END

GDC approach (M1)(A1)

$FV = 13727.85705$

Thus, the amount after 4 years is $\$13700$. (A1)

(ii) The interest

$= 13727.85705 - 10000$ $\qquad I = FV - PV$ (M1)

$= \$3727.857051$

$= \$3730$ (A1)

(b) By financial solver:

N(n) = ?
I% = 8
PV = −10000
PMT(Pmt) = 0
FV = 25000
P / Y(PpY) = 4
C / Y(CpY) = 4
PMT(PmtAt) : END

GDC approach (M1)(A1)

$N = 46.27116989$

The number of years

$= \frac{46.27116989}{4}$

$= 11.56779247$ $\qquad 11.56779247$ (A1)

Thus, the required year is 2036. (A1)

(c) (i) 5% (A1)

(ii) By financial solver:

$\text{N(n)} = 16$
$\text{I\%} = 5$
$\text{PV} = -10000$
$\text{PMT(Pmt)} = 0$
$\text{FV} = ?$
$\text{P / Y(PpY)} = 4$
$\text{C / Y(CpY)} = 4$
$\text{PMT(PmtAt)} : \text{END}$

GDC approach (M1)(A1)

$\text{FV} = 12198.89548$

Thus, the real amount after 4 years is $\$12200$. (A1)

Exercise 1.7

(a) (i) By financial solver:

$N(n)=270$
$I\%=3.3$
$PV=300000$
$PMT(Pmt)=?$
$FV=0$
$P/Y(PpY)=12$
$C/Y(CpY)=12$
$PMT(PmtAt):END$

GDC approach (M1)(A1)

$PMT=-1575.653923$

Thus, the amount of monthly payment is 1580 USD. (A1)

(ii) The total amount

$=(1575.653923)(270)$ $PMT\times N$ (A1)

$=425426.5592$ USD

$=425000$ USD (A1)

(iii) The amount of interest

$=425426.5592-300000$ $I=FV-PV$ (M1)

$=125426.5592$ USD

$=125000$ USD (A1)

(b) (i) By financial solver:

$N(n)=?$
$I\%=3.3$
$PV=300000$
$PMT(Pmt)=-2250$
$FV=0$
$P/Y(PpY)=12$
$C/Y(CpY)=12$
$PMT(PmtAt):END$

GDC approach (M1)(A1)

$N=166.3222392$

Thus, the required number of months is 167. (A1)

(ii) The exact total amount

$=(167)(2250)$ $PMT\times N$ (A1)

$=375750$ USD (A1)

(iii) The amount of interest

$= 375750 - 300000$ — $I = FV - PV$ (M1)

$= 75750$ USD — (A1)

(c) The amount of monthly payment in option 1 is less than that in option 2. — Comparing monthly payment (R1)

Thus, the option 1 is better. — (A1)

Exercise 1.8

(a) (i) $z_1^{\,4}$

$= \left(2\text{cis}\frac{\pi}{8}\right)^4$

$= 16\text{cis}\frac{\pi}{2}$ $\qquad r = 16$ (A1) & $\theta = \frac{\pi}{2}$ (A1)

(ii) The imaginary part of $z_1^{\,4}$

$= 16\sin\frac{\pi}{2}$

$= 16$ $\qquad$ (A1)

(b) (i) $\frac{z_2}{z_1^{\,4}}$

$= \frac{\text{cis}\frac{\pi}{6}}{16\text{cis}\frac{\pi}{2}}$

$= \frac{1}{16}\text{cis}\left(-\frac{\pi}{3}\right)$ $\qquad r = \frac{1}{16}$ (A1) & $\theta = -\frac{\pi}{3}$ (A1)

(ii) $\frac{z_2}{z_1^{\,4}} = \frac{1}{16}e^{-\frac{\pi}{3}i}$ $\qquad$ (A1)

(c) $\arg\left(\frac{z_2}{z_1}\right)^n$

$= n\arg\left(\frac{z_2}{z_1}\right)$

$= n\left(\frac{\pi}{6}-\frac{\pi}{8}\right)$ $\arg\left(\frac{z_2}{z_1}\right)=\arg z_2-\arg z_1$ (M1)

$=\frac{n\pi}{24}$

$\therefore \frac{n\pi}{24}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\cdots$ $N+\frac{\pi}{2}$ (M1)

$\frac{n}{24}=\frac{1}{2},\frac{3}{2},\frac{5}{2},\cdots$

$n=12, 36, 60,\cdots$

Thus, the least value of n is 12. (A1)

Exercise 1.9

V_1

$= 12\cos(bt)$

$= \text{Re}(12e^{bt\text{i}})$ $\text{Re}(z)$ (M1)

V_2

$= 5\cos\left(bt + \frac{\pi}{16}\right)$

$= \text{Re}\left(5e^{\left(bt+\frac{\pi}{16}\right)\text{i}}\right)$

$V_1 + V_2$

$= \text{Re}(12e^{bt\text{i}}) + \text{Re}\left(5e^{\left(bt+\frac{\pi}{16}\right)\text{i}}\right)$

$= \text{Re}\left(12e^{bt\text{i}} + 5e^{\left(bt+\frac{\pi}{16}\right)\text{i}}\right)$ $\text{Re}(z_1) + \text{Re}(z_2) = \text{Re}(z_1 + z_2)$ (M1)

$= \text{Re}\left(e^{bt\text{i}}\left(12 + 5e^{\frac{\pi}{16}\text{i}}\right)\right)$

$= \text{Re}\left(e^{bt\text{i}}(16.93204753e^{0.057641699\text{i}})\right)$

$= \text{Re}(16.93204753e^{bt\text{i}+0.057641699\text{i}})$ $16.93204753e^{bt\text{i}-0.057641699\text{i}}$ (A1)

$= \text{Re}(16.93204753e^{(bt+0.057641699)\text{i}})$

$= 16.93204753\cos(bt + 0.057641699)$

$\therefore V_0 = 16.9$ & $\alpha = 0.0576$ (A1)(A1)

Exercise 1.10

(a) The characteristic polynomial

$= \det(\mathbf{A} - \lambda\mathbf{I})$

$= \begin{vmatrix} 3-\lambda & -12 \\ -2 & 5-\lambda \end{vmatrix}$ $\det(\mathbf{A} - \lambda\mathbf{I})$ (M1)

$= (3-\lambda)(5-\lambda) - (-12)(-2)$

$= 15 - 3\lambda - 5\lambda + \lambda^2 - 24$

$= \lambda^2 - 8\lambda - 9$ (A1)

(b) (i) $\lambda_1 = -1$, $\lambda_2 = 9$ (A1)(A1)

(ii) $\mathbf{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ (A1)(A1)

(c) (i) $\mathbf{P} = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ (A1)

(ii) $\mathbf{D}^n = \begin{pmatrix} (-1)^n & 0 \\ 0 & 9^n \end{pmatrix}$ (A1)

(d) $\mathbf{A}^n$

$= \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$

$= \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^n & 0 \\ 0 & 9^n \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}^{-1}$

$= \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^n & 0 \\ 0 & 9^n \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$ $\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$ (A1)

$= \begin{pmatrix} 3(-1)^n & -2 \cdot 9^n \\ (-1)^n & 9^n \end{pmatrix} \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$ $\begin{pmatrix} 3(-1)^n & -2 \cdot 9^n \\ (-1)^n & 9^n \end{pmatrix}$ (A1)

$= \begin{pmatrix} \frac{3}{5}(-1)^n + \frac{2}{5} \cdot 9^n & \frac{6}{5}(-1)^n - \frac{6}{5} \cdot 9^n \\ \frac{1}{5}(-1)^n - \frac{1}{5} \cdot 9^n & \frac{2}{5}(-1)^n + \frac{3}{5} \cdot 9^n \end{pmatrix}$ (A1)

Exercise 1.11

(a) (i) $\mathbf{A}$

$= \begin{pmatrix} \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$ $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (M1)

$= \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ (A1)

(ii) $\tan\theta = 0$

$\theta = 0$ $\theta = 0$ (A1)

$\mathbf{B}$

$= \begin{pmatrix} \cos 2(0) & \sin 2(0) \\ \sin 2(0) & -\cos 2(0) \end{pmatrix}$ $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ (M1)

$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (A1)

(iii) $\tan\theta = 1$

$\theta = \frac{\pi}{4}$ $\theta = 0$ (A1)

$\mathbf{C}$

$= \begin{pmatrix} \cos 2\left(\frac{\pi}{4}\right) & \sin 2\left(\frac{\pi}{4}\right) \\ \sin 2\left(\frac{\pi}{4}\right) & -\cos 2\left(\frac{\pi}{4}\right) \end{pmatrix}$ $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ (M1)

$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (A1)

(iv) $\mathbf{T}$

$= \mathbf{CBA}$ $\mathbf{CBA}$ (M1)

$= \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ (A1)

(b) (i) $\mathbf{T}^{12}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ (A1)

(ii) $\mathbf{T}^{12}$ indicates that the three movements are repeated for twelve times, and the toy will return to its original position. (A1) (A1)

(c) $\mathbf{T}$

$$=\begin{pmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}$$

$$=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$ $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ (M1)

$$\therefore\begin{cases}\sin\theta=\frac{1}{2}\\ \cos\theta=\frac{\sqrt{3}}{2}\end{cases}$$ System of equations (M1)

$$\tan\theta=\frac{1}{\sqrt{3}}$$ $\tan\theta=\frac{1}{\sqrt{3}}$ (A1)

$$\theta=\frac{\pi}{6}$$

Thus, the required single transformation is an anticlockwise rotation of $\frac{\pi}{6}$ radians about O. (A1)

(d) $|\det(\mathbf{T})|$

$$=\left\|\begin{matrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{matrix}\right\|$$

$$=\left|\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\right|$$

$$=|1|$$

$$=1$$ $|\det(\mathbf{T})|=1$ (A1)

Area of the triangle $\mathrm{U'V'W'}$

$=|\det(\mathbf{T})|\times$Area of the triangle UVW

$=$Area of the triangle UVW Area ratio (R1)

Thus, the claim is correct. (A1)