Chapter 6 Solution

Exercise 19

1. (a) Suppose N is an odd number. M1

N^2 is an odd number and $8N$ is an even number A1

N^2+8N is an odd number

$N^2+8N+10$ is an odd number, which contradicts with the statement $N^2+8N+10$ is an even number.

Thus, if $N^2+8N+10$ is an even number, then N is also an even number. AG

[2]

(b) Let $P=1$ and $Q=-1$. A1

$|P-Q|=|1-(-1)|$

$|P-Q|=|2|$

$|P-Q|=2$

$|P|-|Q|=|1|-|-1|$ M1

$|P|-|Q|=1-1$

$|P|-|Q|=0$

$\therefore |P-Q|>|P|-|Q|$

Thus, $|P-Q|\le|P|-|Q|$ is not always true. AG

[2]

2. (a) Suppose $\frac{1}{x+2}+\frac{x+4}{2}=0$ has real solution. M1

$2(x+2)\left(\frac{1}{x+2}+\frac{x+4}{2}\right)=0$

$2+(x+2)(x+4)=0$ A1

$2+x^2+6x+8=0$

$x^2+6x+10=0$

$\Delta=6^2-4(1)(10)$ A1

$\Delta=-4$

$\Delta<0$, which contradicts with the statement

$\frac{1}{x+2}+\frac{x+4}{2}=0$ has real solution.

Thus, the equation $\frac{1}{x+2}+\frac{x+4}{2}=0$ has no real

solution for all real values of x. AG

[3]

(b) Let $a=2$ and $b=0.5$. A1

$e^{\frac{a}{b}}=e^{\frac{2}{0.5}}$

$e^{\frac{a}{b}}=e^4$

$e^a+e^b=e^2+e^{0.5}$ M1

$e^a+e^b=e^{0.5}(e^{1.5}+1)$

As $e^{0.5}<e^2$ and $e^{1.5}+1<e^2$, $e^{0.5}(e^{1.5}+1)<e^2\cdot e^2$

$\therefore e^{\frac{a}{b}}>e^a+e^b$

Thus, $e^{\frac{a}{b}}\leq e^a+e^b$ is not always true. AG

[2]

3. (a) Let $a=-1$ and $b=2$. A1

$\frac{1}{a^2}=\frac{1}{(-1)^2}$

$\frac{1}{a^2}=1$

$\frac{1}{b^2}=\frac{1}{2^2}$ M1

$\frac{1}{b^2}=\frac{1}{4}$

$\therefore \frac{1}{a^2}>\frac{1}{b^2}$

Thus, $\frac{1}{a^2}<\frac{1}{b^2}$ is not always true. AG

[2]

(b) Suppose $2^x+1,\ 2^{2x}+1,\ 2^{3x}+1,\ \ldots$ is an arithmetic sequence. M1

$(2^{2x}+1)-(2^x+1)=(2^{3x}+1)-(2^{2x}+1)$ A1

$2^{2x}+1-2^x-1=2^{3x}+1-2^{2x}-1$

$2^{2x}-2^x=2^{3x}-2^{2x}$

$2^x-1=2^{2x}-2^x$

$(2^x)^2-2(2^x)+1=0$ A1

$(2^x-1)^2=0$

$2^x-1=0$

$2^x=1$

$x=0$, which contradicts with the statement x is a positive real value.

Thus, the sequence $2^x+1,\ 2^{2x}+1,\ 2^{3x}+1,\ \ldots$ is not an arithmetic sequence for all positive real values of x. AG

[3]

4. (a) Let $x = 0.25$. A1

$\log_2 x = \log_2 0.25$

$\log_2 x = \log_2 2^{-2}$

$\log_2 x = -2$

$\log_4 x = \log_4 0.25$ M1

$\log_4 x = \log_4 4^{-1}$

$\log_4 x = -1$

$\therefore \log_2 x < \log_4 x$

Thus, $\log_2 x \geq \log_4 x$ is not always true. AG

[2]

(b) Suppose $\log x$, $\log x^2$, $\log x^3$, … is a geometric sequence. M1

$$\frac{\log x^2}{\log x} = \frac{\log x^3}{\log x^2}$$ A1

$$\frac{2\log x}{\log x} = \frac{3\log x}{2\log x}$$ A1

$2 = \frac{3}{2}$, which arrives at a contradiction.

Thus, the sequence $\log x$, $\log x^2$, $\log x^3$, … is not a geometric sequence for all positive real values of x. AG

[3]