Exercise 5.1

(a)

$h(2)$

$= g(f(2))$ $\quad g(f(2))$ (M1)

$= g(7)$

$= -1$ $\quad$ (A1)

(b)

$h'(x) = g'(f(x)) \cdot f'(x)$ $\quad g'(f(x)) \cdot f'(x)$ (M1)

$h'(2)$

$= g'(f(2)) \cdot f'(2)$

$= g'(7) \cdot f'(2)$ $\quad g'(7) \cdot f'(2)$ (A1)

$= (2)(1)$

$= 2$ $\quad$ (A1)

(c)

$\alpha(2)$

$= f(2)g(2)$ $\quad f(2)g(2)$ (M1)

$= (7)(1)$

$= 7$ $\quad$ (A1)

(d)

$\alpha'(x) = f'(x)g(x) + f(x)g'(x)$ $\quad$ Product rule (M1)

$\alpha'(2)$

$= f'(2)g(2) + f(2)g'(2)$ $\quad f'(2)g(2) + f(2)g'(2)$ (A1)

$= (1)(1) + (7)(0)$

$= 1$ $\quad$ (A1)

(e)

$\beta(7)$

$= \frac{f(7)}{g(7)}$ $\quad \frac{f(7)}{g(7)}$ (M1)

$= \frac{2}{-1}$

$= -2$ $\quad$ (A1)

(f) $\beta'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ Quotient rule (M1)

$\beta'(7)$

$= \dfrac{f'(7)g(7) - f(7)g'(7)}{(g(7))^2}$ $\dfrac{f'(7)g(7) - f(7)g'(7)}{(g(7))^2}$ (A1)

$= \dfrac{(0)(-1) - (2)(2)}{(-1)^2}$

$= -4$ (A1)

Exercise 5.2

(a) $f'(x)$

$$=\frac{\left(\frac{\mathrm{d}}{\mathrm{d}x}(6x+24)\right)(x^2+3x)-(6x+24)\left(\frac{\mathrm{d}}{\mathrm{d}x}(x^2+3x)\right)}{(x^2+3x)^2}$$ Quotient rule (M1)

$$=\frac{(6)(x^2+3x)-(6x+24)(2x+3)}{(x^2+3x)^2}$$ 6 (A1) & $2x+3$ (A1)

$$=\frac{6x^2+18x-(12x^2+66x+72)}{(x^2+3x)^2}$$ $12x^2+66x+72$ (M1)

$$=\frac{-6x^2-48x-72}{(x^2+3x)^2}$$ $-6x^2-48x-72$ (A1)

$$=\frac{-6(x^2+8x+12)}{(x^2+3x)^2}$$

$$=-\frac{6(x+2)(x+6)}{(x^2+3x)^2}$$ (AG)

(b) $f'(x)=0$

$$\therefore -\frac{6(x+2)(x+6)}{(x^2+3x)^2}=0$$ $-\frac{6(x+2)(x+6)}{(x^2+3x)^2}=0$ (M1)

$(x+2)(x+6)=0$ — $(x+2)(x+6)=0$ (A1)

$x+2=0$ or $x+6=0$

$x=-2$ or $x=-6$ (*Rejected*) — $x=-2$ (A1)

$f(-2)$

$$=\frac{6(-2)+24}{(-2)^2+3(-2)}$$ $x=-2$ (M1)

$=-6$

Thus, the coordinates of A are $(-2,-6)$. (A1)

Exercise 5.3

(a) $f(1)$

$=-2e^{-1}$

$=-\frac{2}{e}$ $\quad -\frac{2}{e}$ (A1)

$f'(x)$

$=-2(e^{-x})(-1)$ $\quad$ Chain rule (M1)

$=2e^{-x}$

The slope of the tangent

$=f'(1)$ $\quad f'(1)$ (M1)

$=2e^{-1}$

$=\frac{2}{e}$ $\quad \frac{2}{e}$ (A1)

The equation of the tangent:

$y-\left(-\frac{2}{e}\right)=\frac{2}{e}(x-1)$ $\quad y-y_1=m(x-x_1)$ (M1)

$y+\frac{2}{e}=\frac{2}{e}x-\frac{2}{e}$

$ey+2=2x-2$

$2x-ey-4=0$ $\quad$ (A1)

(b) The slope of the normal

$=-1\div\frac{2}{e}$

$=-\frac{e}{2}$ $\quad -\frac{e}{2}$ (A1)

The equation of the normal:

$y-\left(-\frac{2}{e}\right)=-\frac{e}{2}(x-1)$ $\quad y-y_1=m(x-x_1)$ (M1)

$y+\frac{2}{e}=-\frac{e}{2}x+\frac{e}{2}$

$2ey+4=-e^2x+e^2$

$e^2x+2ey+(4-e^2)=0$ $\quad$ (A1)

Exercise 5.4

(a) $Q = t^3 - 12t^2 + 36t$

$\frac{dQ}{dt}$

$= 3t^2 - 12(2t) + 36(1)$ — $3t^2$, $2t$ & 1 (A1)

$= 3t^2 - 24t + 36$

$\frac{dQ}{dt} = 0$ — $\frac{dQ}{dt} = 0$ (M1)

$3t^2 - 24t + 36 = 0$

$t^2 - 8t + 12 = 0$

$(t-2)(t-6) = 0$

$t - 2 = 0$ or $t - 6 = 0$

$t = 2$ or $t = 6$ — 2 & 6 (A1)

By the first derivative test, — Checking $\frac{dQ}{dt}$ (M1)

t	$t < 2$	$t = 2$	$2 < t < 6$	$t = 6$	$t > 6$
$\frac{dQ}{dt}$	$+$	0	$-$	0	$+$

Thus, Q attains its maximum at $t = 2$. — (A1)

(b) 32 — (A1)

(c) $Q = t^3 - 12t^2 + \alpha t$

$\frac{dQ}{dt} = \frac{d}{dt}(t^3) - \frac{d}{dt}(12t^2) + \frac{d}{dt}(\alpha t)$ — Implicit differentiation (M1)

$\frac{dQ}{dt} = 3t^2 - 12(2t) + \left(\frac{d\alpha}{dt}\right)(t) + (\alpha)(1)$ — $3t^2, 12(2t)$ (A1) & Product rule (A1)

$\frac{dQ}{dt} = 3t^2 - 24t + t\frac{d\alpha}{dt} + \alpha$

The required rate of change

$= 3(1)^2 - 24(1) + (1)(-0.25) + 18$ — $\frac{dQ}{dt}$ (A1)

$= -3.25$ — (A1)

Exercise 5.5

(a) By considering the graph of $y=6\sin t-t^3\cos t$, the coordinates of the maximum point are $(3.7435506, 39.843756)$. GDC approach (M1)

Thus, the maximum distance is 39.8 cm. (A1)

(b) The particle first changes direction at 3.7435506 s. 1.5707983 s (A1)

By considering the graph of $y=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}}{\mathrm{d}t}(6\sin t-t^3\cos t)\right)$, the graph passes through the point $(3.7435506, -68.94427)$. GDC approach (M1)

Thus, the acceleration is -68.9 cms^{-2}. (A1)

Exercise 5.6

$f(x)$

$=\int \cos 2x \sin^3 2x\,\mathrm{d}x$ Indefinite integral (M1)

Let $u=\sin 2x$.

$\frac{\mathrm{d}u}{\mathrm{d}x}=2\cos 2x \Rightarrow \frac{1}{2}\mathrm{d}u=\cos 2x\,\mathrm{d}x$ Substitution (A1)

$=\int u^3\cdot\frac{1}{2}\mathrm{d}u$ $\int u^3\cdot\frac{1}{2}\mathrm{d}u$ (A1)

$=\frac{1}{2}\left(\frac{1}{4}u^4\right)+C$

$=\frac{1}{8}u^4+C$

$\therefore f(x)=\frac{1}{8}\sin^4 2x+C$ $\frac{1}{8}\sin^4 2x+C$ (A1)

$3=\frac{1}{8}\left(\sin\left(2\left(\frac{\pi}{2}\right)\right)\right)^4+C$ $x=\frac{\pi}{2}$ & $y=3$ (M1)

$3=\frac{1}{8}(0)+C$

$C=3$

$\therefore f(x)=\frac{1}{8}\sin^4 2x+3$ (A1)

Exercise 5.7

The area of R

$= \int_0^1 f(x)\mathrm{d}x$ — Definite integral (M1)

$= \int_0^1 \sqrt{\pi x+1}\mathrm{d}x$

Let $u = \pi x + 1$.

$\dfrac{\mathrm{d}u}{\mathrm{d}x} = \pi \Rightarrow \dfrac{1}{\pi}\mathrm{d}u = \mathrm{d}x$

$x = 1 \Rightarrow u = \pi(1)+1 = \pi+1$

$x = 0 \Rightarrow u = \pi(0)+1 = 1$

Substitution (A1)

$= \int_1^{\pi+1} \sqrt{u}\cdot\dfrac{1}{\pi}\mathrm{d}u$ — $\int_1^{\pi+1} \sqrt{u}\cdot\dfrac{1}{\pi}\mathrm{d}u$ (A1)

$= \dfrac{1}{\pi}\int_1^{\pi+1} u^{\frac{1}{2}}\mathrm{d}u$

$= \dfrac{1}{\pi}\left[\dfrac{2}{3}u^{\frac{3}{2}}\right]_1^{\pi+1}$ — $\dfrac{2}{3}u^{\frac{3}{2}}$ (A1)

$= \dfrac{1}{\pi}\left(\dfrac{2}{3}(\pi+1)^{\frac{3}{2}} - \dfrac{2}{3}(1)^{\frac{3}{2}}\right)$

$= \dfrac{2}{3\pi}\left((\pi+1)^{\frac{3}{2}} - 1\right)$ — (A1)

Exercise 5.8

(a) The velocity

$= v(3)$ — $v(3)$ (M1)

$= 3^2 e^{-0.5(3)}$

$= 2.008171441$ ms^{-1}

$= 2.01$ ms^{-1} — (A1)

(b) $v(t) = 1$ — $v(t) = 1$ (M1)

$t^2 e^{-0.5t} = 1$

$t^2 e^{-0.5t} - 1 = 0$

By considering the graph of $y = t^2 e^{-0.5t} - 1$, the horizontal intercepts are 1.4296118 and 8.6131695. — GDC approach (M1)

$\therefore t = 1.43$ or $t = 8.61$ — (A1)

(c) The total distance travelled

$= \int_0^{10} |v(t)| \, \mathrm{d}t$ — $\int_{t_1}^{t_2} |v(t)| \, \mathrm{d}t$ (M1)

$= \int_0^{10} |t^2 e^{-0.5t}| \, \mathrm{d}t$ — $\int_0^{10} |t^2 e^{-0.5t}| \, \mathrm{d}t$ (M1)

$= 14.00556769$ m

$= 14.0$ m — (A1)

(d) $a(t)$

$= v'(t)$ — $v'(t)$ (M1)

$= (2t)(e^{-0.5t}) + (t^2)(e^{-0.5t})(-0.5)$ — Product rule (M1)

$= 2te^{-0.5t} - 0.5t^2 e^{-0.5t}$ — (A1)

(e) By considering the graphs of $y = t^2 e^{-0.5t}$ and $y = \frac{\mathrm{d}}{\mathrm{d}t}(t^2 e^{-0.5t})$, at least one graph is below the horizontal axis for $4 \le t \le 10$. — GDC approach (M1)

$\therefore 4 \le t \le 10$ — 4 (A1) & 10 (A1)