Solución de Práctica de Prueba 1 de AE NS Set 4

Sección A

1. (a) El área de la region sombreada

$=\frac{1}{2}(20)^2(1,5)$ (A1) por sustitución

$=300\ \text{cm}^2$ A1

[2]

(b) La longitud del arco ABC

$=(20)(1,5)$ (A1) por sustitución

$=30\ \text{cm}$ A1

[2]

(c) El perimetro requerido

$=2\pi(20)-30+20+20$ (M1) por enfoque válido

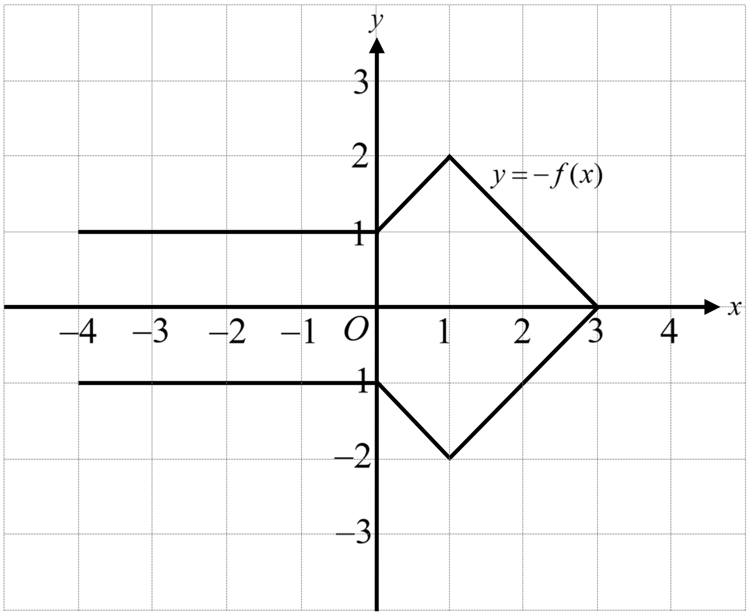
$=(40\pi+10)\ \text{cm}$ A1

[2]

2. (a) Por los puntos de intersección con el eje x e y correctos A1

Por dos puntos correctos $(-4, 1)$ y $(1, 2)$ A1

[2]



(b) $p = 2$ A2

$q = -1$ A2

[4]

3. (a) $\log_4 64$

$= \log_4 4^3$ (A1) por enfoque correcto

$= 3$ A1

[2]

(b) $\log_{12} 36 + \log_{12} 4$

$= \log_{12} 144$ (A1) por enfoque correcto

$= \log_{12} 12^2$

$= 2$ A1

[2]

(c) $\log_2 11 - \log_2 88$

$= \log_2 \frac{1}{8}$ (A1) por enfoque correcto

$= \log_2 2^{-3}$

$= -3$ A1

[2]

4. (a) $a = 2(-\text{sen}\,\pi t)(\pi) + 0$ — (A1) por derivadas correctas

$a = -2\pi\,\text{sen}\,\pi t$ — A1

[2]

(b) $s = \int (2\cos \pi t + \pi)\mathrm{d}t$ — (M1) por integral indefinida

$s = \int 2\cos \pi t \mathrm{d}t + \int \pi \mathrm{d}t$

$$\boxed{\begin{array}{l}\text{Sea } u = \pi t \\ \dfrac{\mathrm{d}u}{\mathrm{d}t} = \pi \Rightarrow \dfrac{1}{\pi}\mathrm{d}u = \mathrm{d}t\end{array}}$$

(A1) por sustitución

$s = \int \frac{2}{\pi}\cos u \mathrm{d}u + \int \pi \mathrm{d}t$

$s = \frac{2}{\pi}\text{sen}\,u + \pi t + C$ — A1

$s = \frac{2}{\pi}\text{sen}\,\pi t + \pi t + C$

$\therefore -3 = \frac{2}{\pi}\text{sen}\,0 + 0 + C$ — (M1) por sustitución

$C = -3$

$\therefore s = \frac{2}{\pi}\text{sen}\,\pi t + \pi t - 3$ — A1

[5]

5. (a) $1 < D < 5$ — A1

[1]

(b) 6 horas — A1

[1]

(c) (i) La media requerida

$= 10,5 + 1,5$ — (M1) por enfoque válido

$= 12$ — A1

(ii) La varianza requerida

$= 2^2$ — (M1)(A1) por enfoque correcto

$= 4$ — A1

[5]

6. $$\lim_{x\to 0}\frac{1+3x-\cos\frac{\pi}{3}x}{\ln(1+x)}$$

$$=\lim_{x\to 0}\frac{0+3-\left(-\operatorname{sen}\frac{\pi}{3}x\right)\left(\frac{\pi}{3}\right)}{\left(\frac{1}{x+1}\right)(1)}\left(\because\frac{0}{0}\right)$$ M1A2

$$=\lim_{x\to 0}(x+1)\left(3+\frac{\pi}{3}\operatorname{sen}\frac{\pi}{3}x\right)$$

$$=(0+1)\left(3+\frac{\pi}{3}\operatorname{sen}\frac{\pi}{3}(0)\right)$$ M1

$$=3$$ A1

[5]

7. $$\tan x+\cot x+\frac{4\sqrt{3}}{3}=0$$

$$\tan x+\cot x=-\frac{4\sqrt{3}}{3}$$

$$\frac{\operatorname{sen}x}{\cos x}+\frac{\cos x}{\operatorname{sen}x}=-\frac{4\sqrt{3}}{3}$$ (A1) por sustitución

$$\frac{\operatorname{sen}^2 x+\cos^2 x}{\operatorname{sen}x\cos x}=-\frac{4\sqrt{3}}{3}$$

$$1=-\frac{4\sqrt{3}}{3}\operatorname{sen}x\cos x$$ (M1) por enfoque válido

$$-\sqrt{3}=2(2\operatorname{sen}x\cos x)$$

$$\operatorname{sen}2x=-\frac{\sqrt{3}}{2}$$ A1

$2x=\pi+\frac{\pi}{3}$ o $2x=2\pi-\frac{\pi}{3}$

$x=\frac{2\pi}{3}$ o $x=\frac{5\pi}{6}$ A2

[5]

8. (a) $L_1 : \begin{cases} x = 17 + 5t \\ y = 1 - 2t \\ z = 10 + 3t \end{cases}$

$(17 + 5t) - 8 = 3 - (10 + 3t)$ (M1) por ecuación

$9 + 5t = -7 - 3t$

$16 = -8t$

$t = -2$ A1

$\therefore \begin{cases} x = 17 + 5(-2) = 7 \\ y = 1 - 2(-2) = 5 \\ z = 10 + 3(-2) = 4 \end{cases}$ (M1) por sustitución

Por tanto, las coordenadas de P son $(7, 5, 4)$. A1

[4]

(b) $\overrightarrow{\mathrm{RQ}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$\therefore \overrightarrow{\mathrm{OQ}} - \overrightarrow{\mathrm{OR}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ (M1) por enfoque válido

$((17 + 5t)\mathbf{i} + (1 - 2t)\mathbf{j} + (10 + 3t)\mathbf{k}) - (3\mathbf{i} + 5\mathbf{k})$

$= -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$17 + 5t - 3 = -1$

$5t = -15$

$t = -3$ (A1) por valor correcto

$\therefore \begin{cases} x = 17 + 5(-3) = 2 \\ y = 1 - 2(-3) = 7 \\ z = 10 + 3(-3) = 1 \end{cases}$ (M1) por sustitución

Por tanto, las coordenadas de Q son $(2, 7, 1)$. A1

[4]

9. Cuando $n=1$,

$5-21(1)+4^1=-12$

$5-21(1)+4^1=3(-4)$ A1

Por lo tanto, el enunciado es cierto cuando $n=1$.

Suponemos que es cierto para $n=k$. M1

$5-21k+4^k=3M$, donde $M\in\mathbb{Z}$.

Cuando $n=k+1$,

$5-21(k+1)+4^{k+1}$

$=5-21k-21+4(4^k)$ M1

$=-16-21k+4(3M+21k-5)$ A1

$=-16-21k+12M+84k-20$

$=12M+63k-36$ M1

$=3(4M+21k-12)$, donde $4M+21k-12\in\mathbb{Z}$. A1

Por lo tanto, el enunciado es cierto cuando $n=k+1$.

Por lo tanto, el enunciado es cierto para todo $n\in\mathbb{Z}^+$. R1

[7]

Sección B

10. (a) (i) La probabilidad requerida

$= \frac{3}{n}$ A1

(ii) La probabilidad requerida

$= \left(\frac{n-3}{n}\right)\left(\frac{n-4}{n-1}\right)\left(\frac{3}{n-2}\right)$ (A1) por enfoque correcto

$= \frac{3(n-3)(n-4)}{n(n-1)(n-2)}$ A1

[3]

(b) La probabilidad requerida

$= \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$ (A1) por enfoque correcto

$= \frac{1}{8}$ A1

[2]

(c) El juego es justo si la ganancia esperada es cero, lo que equivale a que la cantidad esperada de dinero recuperado sea $10\$$.R1

$\therefore \left(\frac{3}{10}\right)(10) + \left(\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\right)(10)$

$+ \left(\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\right)(25x) + \left(\frac{1}{8}\right)(21x)$ M1A2

$+ \left(1 - \frac{3}{10} - \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) - \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right) - \frac{1}{8}\right)(0) = 10$

$3 + \frac{7}{3} + \frac{35}{8}x + \frac{21}{8}x = 10$ M1A1

$7x = \frac{14}{3}$ A1

$x = \frac{2}{3}$ AG

[7]

11. (a) $z^{20}=1$

$z^{20}=\cos 0+\mathrm{i}\,\mathrm{sen}\,0$ A1

$z=\cos\left(\dfrac{0+2k\pi}{20}\right)+\mathrm{i}\,\mathrm{sen}\left(\dfrac{0+2k\pi}{20}\right)$ M1

$(k=0,1,2,\cdots,18,19)$

$z=\cos 0+\mathrm{i}\,\mathrm{sen}\,0$, $z=\cos\dfrac{\pi}{10}+\mathrm{i}\,\mathrm{sen}\dfrac{\pi}{10}$,

$z=\cos\dfrac{\pi}{5}+\mathrm{i}\,\mathrm{sen}\dfrac{\pi}{5}$, …,

$z=\cos\dfrac{9\pi}{5}+\mathrm{i}\,\mathrm{sen}\dfrac{9\pi}{5}$ o $z=\cos\dfrac{19\pi}{10}+\mathrm{i}\,\mathrm{sen}\dfrac{19\pi}{10}$ (A1) por valores correctos

$-\dfrac{\pi}{2}\le\arg(z)\le 0$

$\therefore z=\mathrm{cis}0$, $z=\mathrm{cis}\left(-\dfrac{\pi}{2}\right)$, $z=\mathrm{cis}\left(-\dfrac{2\pi}{5}\right)$,

$z=\mathrm{cis}\left(-\dfrac{3\pi}{10}\right)$, $z=\mathrm{cis}\left(-\dfrac{\pi}{5}\right)$ o $z=\mathrm{cis}\left(-\dfrac{\pi}{10}\right)$ A3

[6]

(b) $1+\cos\left(-\dfrac{2\pi}{5}\right)+\cos\left(-\dfrac{3\pi}{10}\right)$
$+\cos\left(-\dfrac{\pi}{5}\right)+\cos\left(-\dfrac{\pi}{10}\right)$ A1

[1]

(c) $\operatorname{Im} S$

$$= -1 + \operatorname{sen}\left(-\frac{2\pi}{5}\right) + \operatorname{sen}\left(-\frac{3\pi}{10}\right)$$

$$+ \operatorname{sen}\left(-\frac{\pi}{5}\right) + \operatorname{sen}\left(-\frac{\pi}{10}\right)$$ A1

$$= -1 - \operatorname{sen}\frac{2\pi}{5} - \operatorname{sen}\frac{3\pi}{10} - \operatorname{sen}\frac{\pi}{5} - \operatorname{sen}\frac{\pi}{10}$$ M1

$$= -1 - \cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$$

$$- \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{\pi}{10}\right)$$ A1

$$= -1 - \cos\frac{\pi}{10} - \cos\frac{\pi}{5} - \cos\frac{3\pi}{10} - \cos\frac{4\pi}{10}$$ M1

$$= -\left(\begin{array}{l} 1 + \cos\left(-\frac{2\pi}{5}\right) + \cos\left(-\frac{3\pi}{10}\right) \\ + \cos\left(-\frac{\pi}{5}\right) + \cos\left(-\frac{\pi}{10}\right) \end{array}\right)$$ A1

$$= -\operatorname{Re} S$$

$$\therefore \frac{\operatorname{Re} S}{\operatorname{Im} S} = -1$$ AG

[5]

(d) (i) $\cos\left(-\frac{\pi}{5}\right)$

$$= \cos\frac{\pi}{5}$$

$$= 2\cos^2\frac{\pi}{10} - 1$$ (A1) por sustitución

$$= 2\left(\frac{\sqrt{10 + 2\sqrt{5}}}{4}\right)^2 - 1$$

$$= \frac{10 + 2\sqrt{5}}{8} - 1$$ (M1) por enfoque válido

$$= \frac{10 + 2\sqrt{5} - 8}{8}$$

$$= \frac{1 + \sqrt{5}}{4}$$ A1

(ii) $\cos\left(-\frac{2\pi}{5}\right)$

$=2\cos^2\left(-\frac{\pi}{5}\right)-1$ (A1) por sustitución

$=2\left(\frac{1+\sqrt{5}}{4}\right)^2-1$

$=\frac{1+2\sqrt{5}+5}{8}-1$ (M1) por enfoque válido

$=\frac{6+2\sqrt{5}-8}{8}$

$=\frac{\sqrt{5}-1}{4}$ A1

[6]

(e) $\operatorname{Im} S$

$=-\operatorname{Re} S$

$=-\begin{pmatrix} 1+\cos\left(-\frac{2\pi}{5}\right)+\cos\left(-\frac{3\pi}{10}\right) \\ +\cos\left(-\frac{\pi}{5}\right)+\cos\left(-\frac{\pi}{10}\right) \end{pmatrix}$ M1

$=-\left(1+\frac{\sqrt{5}-1}{4}+\frac{\sqrt{10-2\sqrt{5}}}{4}+\frac{1+\sqrt{5}}{4}+\frac{\sqrt{10+2\sqrt{5}}}{4}\right)$ A1

$=-\left(1+\frac{\sqrt{5}}{2}+\frac{\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}\right)$

$=-\left(\frac{4}{4}+\frac{2\sqrt{5}}{4}+\frac{\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}\right)$

$=-\frac{4+2\sqrt{5}+\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}$ AG

[2]

12. (a) (i) $f(x)=g(x)$

$\therefore \operatorname{sen} 2\pi y=-\operatorname{sen} \pi y$ M1

$2\operatorname{sen} \pi y\cos \pi y+\operatorname{sen} \pi y=0$ A1

$\operatorname{sen} \pi y(2\cos \pi y+1)=0$ A1

$\operatorname{sen} \pi y=0$ o $\cos \pi y=-\dfrac{1}{2}$

$\pi y=0$ o $\pi y=\dfrac{2\pi}{3}$ A1

$y=0$ (*Rechazada*) o $y=\dfrac{2}{3}$

$\therefore r=\dfrac{2}{3}$ AG

(ii) El área de la región

$=\int_{\frac{2}{3}}^{1}(g(y)-f(y))\mathrm{d}y$ A1

$=\int_{\frac{2}{3}}^{1}(-\operatorname{sen} \pi y-\operatorname{sen} 2\pi y)\mathrm{d}y$

$=\left[\dfrac{1}{\pi}\cos \pi y+\dfrac{1}{2\pi}\cos 2\pi y\right]_{\frac{2}{3}}^{1}$ A1

$=\left(\dfrac{1}{\pi}\cos \pi(1)+\dfrac{1}{2\pi}\cos 2\pi(1)\right)$

$-\left(\dfrac{1}{\pi}\cos \pi\left(\dfrac{2}{3}\right)+\dfrac{1}{2\pi}\cos 2\pi\left(\dfrac{2}{3}\right)\right)$ M1

$=\left(-\dfrac{1}{\pi}+\dfrac{1}{2\pi}\right)-\left(\dfrac{1}{\pi}\left(-\dfrac{1}{2}\right)+\dfrac{1}{2\pi}\left(-\dfrac{1}{2}\right)\right)$ A1

$=-\dfrac{1}{2\pi}-\left(-\dfrac{1}{2\pi}-\dfrac{1}{4\pi}\right)$ M1

$=-\dfrac{1}{2\pi}+\dfrac{1}{2\pi}+\dfrac{1}{4\pi}$

$=\dfrac{1}{4\pi}$ AG

[9]

(b) $a\,\text{sen}\,2\pi\left(\frac{3}{4}\right)=-\frac{\sqrt{2}}{2}$ (M1) por sustitución

$-a=-\frac{\sqrt{2}}{2}$ A1

$a=\frac{\sqrt{2}}{2}$ A1

[3]

(c) $f(x)=g(x)$

$\therefore a\,\text{sen}\,2\pi y=-\text{sen}\,\pi y$ M1

$2a\,\text{sen}\,\pi y\cos\pi y+\text{sen}\,\pi y=0$ A1

$\text{sen}\,\pi y(2a\cos\pi y+1)=0$ A1

$2a\cos\pi y+1=0$ M1

$2a\cos\pi y=-1$

$\cos\pi y=-\frac{1}{2a}$ A1

$\therefore \text{sen}\,\pi y$

$=\sqrt{1-\cos^2\pi y}$ A1

$=\sqrt{1-\left(-\frac{1}{2a}\right)^2}$

$=\sqrt{1-\frac{1}{4a^2}}$ M1

$=\sqrt{\frac{4a^2-1}{4a^2}}$ A1

$\pi y=\text{arcsen}\sqrt{\frac{4a^2-1}{4a^2}}$ M1

$\therefore r=\frac{1}{\pi}\text{arcsen}\sqrt{\frac{4a^2-1}{4a^2}}$ AG

[9]