AA HL Practice Set 4 Paper 1 Solution

Section A

1. (a) The area of the shaded region

$=\frac{1}{2}(20)^2(1.5)$ (A1) for substitution

$=300\text{ cm}^2$ A1

[2]

(b) The arc length ABC

$=(20)(1.5)$ (A1) for substitution

$=30\text{ cm}$ A1

[2]

(c) The required perimeter

$=2\pi(20)-30+20+20$ (M1) for valid approach

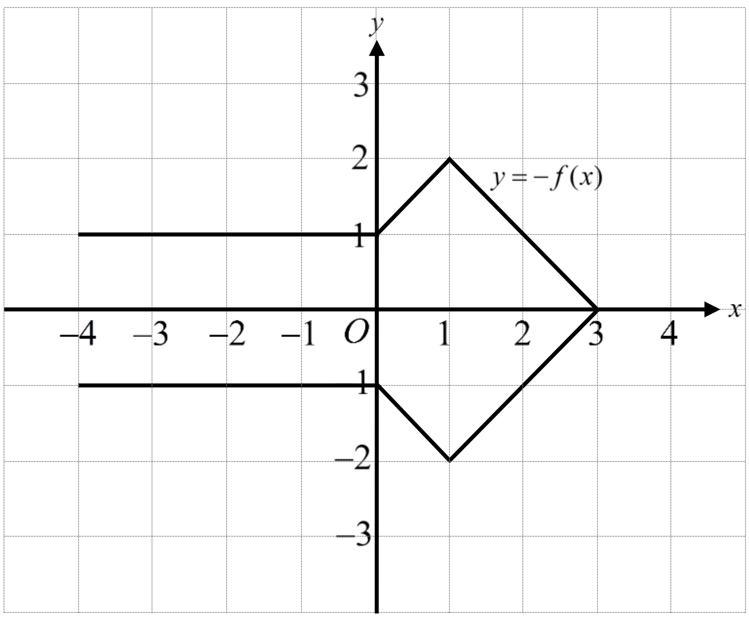
$=(40\pi+10)\text{ cm}$ A1

[2]

2. (a) For correct x-intercept and y-intercept A1

For two correct points $(-4, 1)$ and $(1, 2)$ A1

[2]



(b) $p=2$ A2

$q=-1$ A2

[4]

3. (a) $\log_4 64$

$= \log_4 4^3$ (A1) for correct approach

$= 3$ A1

[2]

(b) $\log_{12} 36 + \log_{12} 4$

$= \log_{12} 144$ (A1) for correct approach

$= \log_{12} 12^2$

$= 2$ A1

[2]

(c) $\log_2 11 - \log_2 88$

$= \log_2 \frac{1}{8}$ (A1) for correct approach

$= \log_2 2^{-3}$

$= -3$ A1

[2]

4. (a) $a = 2(-\sin \pi t)(\pi) + 0$ (A1) for correct derivatives

$a = -2\pi \sin \pi t$ A1

[2]

(b) $s = \int (2\cos \pi t + \pi) \mathrm{d}t$ (M1) for indefinite integral

$s = \int 2\cos \pi t \mathrm{d}t + \int \pi \mathrm{d}t$

Let $u = \pi t$
$\frac{\mathrm{d}u}{\mathrm{d}t} = \pi \Rightarrow \frac{1}{\pi}\mathrm{d}u = \mathrm{d}t$

(A1) for substitution

$s = \int \frac{2}{\pi} \cos u \mathrm{d}u + \int \pi \mathrm{d}t$

$s = \frac{2}{\pi} \sin u + \pi t + C$ A1

$s = \frac{2}{\pi} \sin \pi t + \pi t + C$

$\therefore -3 = \frac{2}{\pi} \sin 0 + 0 + C$ (M1) for substitution

$C = -3$

$\therefore s = \frac{2}{\pi} \sin \pi t + \pi t - 3$ A1

[5]

5. (a) $1 < D < 5$ A1

[1]

(b) 6 hours A1

[1]

(c) (i) The required mean

$= 10.5 + 1.5$ (M1) for valid approach

$= 12$ A1

(ii) The required variance

$= 2^2$ (M1)(A1) for correct approach

$= 4$ A1

[5]

6. $\lim_{x \to 0} \dfrac{1 + 3x - \cos\frac{\pi}{3}x}{\ln(1+x)}$

$= \lim_{x \to 0} \dfrac{0 + 3 - \left(-\sin\frac{\pi}{3}x\right)\left(\frac{\pi}{3}\right)}{\left(\frac{1}{x+1}\right)(1)} \left(\because \frac{0}{0}\right)$ M1A2

$= \lim_{x \to 0}(x+1)\left(3 + \frac{\pi}{3}\sin\frac{\pi}{3}x\right)$

$= (0+1)\left(3 + \frac{\pi}{3}\sin\frac{\pi}{3}(0)\right)$ M1

$= 3$ A1

[5]

7. $\tan x + \cot x + \frac{4\sqrt{3}}{3} = 0$

$\tan x + \cot x = -\frac{4\sqrt{3}}{3}$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -\frac{4\sqrt{3}}{3}$ (A1) for substitution

$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -\frac{4\sqrt{3}}{3}$

$1 = -\frac{4\sqrt{3}}{3} \sin x \cos x$ (M1) for valid approach

$-\sqrt{3} = 2(2\sin x \cos x)$

$\sin 2x = -\frac{\sqrt{3}}{2}$ A1

$2x = \pi + \frac{\pi}{3}$ or $2x = 2\pi - \frac{\pi}{3}$

$x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{6}$ A2

[5]

8. (a) $L_1 : \begin{cases} x = 17 + 5t \\ y = 1 - 2t \\ z = 10 + 3t \end{cases}$

$(17 + 5t) - 8 = 3 - (10 + 3t)$ (M1) for setting equation

$9 + 5t = -7 - 3t$

$16 = -8t$

$t = -2$ A1

$\therefore \begin{cases} x = 17 + 5(-2) = 7 \\ y = 1 - 2(-2) = 5 \\ z = 10 + 3(-2) = 4 \end{cases}$ (M1) for substitution

Thus, the coordinates of P are $(7, 5, 4)$. A1

[4]

(b) $\overrightarrow{\text{RQ}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$\therefore \overrightarrow{\text{OQ}} - \overrightarrow{\text{OR}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ (M1) for valid approach

$((17 + 5t)\mathbf{i} + (1 - 2t)\mathbf{j} + (10 + 3t)\mathbf{k}) - (3\mathbf{i} + 5\mathbf{k})$

$= -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$17 + 5t - 3 = -1$

$5t = -15$

$t = -3$ (A1) for correct value

$\therefore \begin{cases} x = 17 + 5(-3) = 2 \\ y = 1 - 2(-3) = 7 \\ z = 10 + 3(-3) = 1 \end{cases}$ (M1) for substitution

Thus, the coordinates of Q are $(2, 7, 1)$. A1

[4]

9. When $n=1$,

$5-21(1)+4^1=-12$

$5-21(1)+4^1=3(-4)$ A1

Thus, the statement is true when $n=1$.

Assume that the statement is true when $n=k$. M1

$5-21k+4^k=3M$, where $M\in\mathbb{Z}$.

When $n=k+1$,

$5-21(k+1)+4^{k+1}$

$=5-21k-21+4(4^k)$ M1

$=-16-21k+4(3M+21k-5)$ A1

$=-16-21k+12M+84k-20$

$=12M+63k-36$ M1

$=3(4M+21k-12)$, where $4M+21k-12\in\mathbb{Z}$. A1

Thus, the statement is true when $n=k+1$.

Therefore, the statement is true for all $n\in\mathbb{Z}^+$. R1

[7]

Section B

10. (a) (i) The required probability

$= \frac{3}{n}$ A1

(ii) The required probability

$= \left(\frac{n-3}{n}\right)\left(\frac{n-4}{n-1}\right)\left(\frac{3}{n-2}\right)$ (A1) for correct approach

$= \frac{3(n-3)(n-4)}{n(n-1)(n-2)}$ A1

[3]

(b) The required probability

$= \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$ (A1) for correct approach

$= \frac{1}{8}$ A1

[2]

(c) The game is fair if the expected gain is zero, which is equivalent to the expected amount of money earns back equals to $10. R1

$\therefore \left(\frac{3}{10}\right)(10) + \left(\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\right)(10)$

$+ \left(\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\right)(25x) + \left(\frac{1}{8}\right)(21x)$ M1A2

$+ \left(1 - \frac{3}{10} - \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) - \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right) - \frac{1}{8}\right)(0) = 10$

$3 + \frac{7}{3} + \frac{35}{8}x + \frac{21}{8}x = 10$ M1A1

$7x = \frac{14}{3}$ A1

$x = \frac{2}{3}$ AG

[7]

11. (a)

$z^{20} = 1$

$z^{20} = \cos 0 + \mathrm{i}\sin 0$ A1

$z = \cos\left(\frac{0+2k\pi}{20}\right) + \mathrm{i}\sin\left(\frac{0+2k\pi}{20}\right)$ M1

$(k = 0, 1, 2, \cdots, 18, 19)$

$z = \cos 0 + \mathrm{i}\sin 0$, $z = \cos\frac{\pi}{10} + \mathrm{i}\sin\frac{\pi}{10}$,

$z = \cos\frac{\pi}{5} + \mathrm{i}\sin\frac{\pi}{5}$, …,

$z = \cos\frac{9\pi}{5} + \mathrm{i}\sin\frac{9\pi}{5}$ or $z = \cos\frac{19\pi}{10} + \mathrm{i}\sin\frac{19\pi}{10}$ (A1) for correct values

$-\frac{\pi}{2} \le \arg(z) \le 0$

$\therefore z = \mathrm{cis}0$, $z = \mathrm{cis}\left(-\frac{\pi}{2}\right)$, $z = \mathrm{cis}\left(-\frac{2\pi}{5}\right)$,

$z = \mathrm{cis}\left(-\frac{3\pi}{10}\right)$, $z = \mathrm{cis}\left(-\frac{\pi}{5}\right)$ or $z = \mathrm{cis}\left(-\frac{\pi}{10}\right)$ A3

[6]

(b)

$1 + \cos\left(-\frac{2\pi}{5}\right) + \cos\left(-\frac{3\pi}{10}\right)$

$+ \cos\left(-\frac{\pi}{5}\right) + \cos\left(-\frac{\pi}{10}\right)$ A1

[1]

(c)

$\mathrm{Im}\, S$

$= -1 + \sin\left(-\frac{2\pi}{5}\right) + \sin\left(-\frac{3\pi}{10}\right)$

$+ \sin\left(-\frac{\pi}{5}\right) + \sin\left(-\frac{\pi}{10}\right)$ A1

$= -1 - \sin\frac{2\pi}{5} - \sin\frac{3\pi}{10} - \sin\frac{\pi}{5} - \sin\frac{\pi}{10}$ M1

$= -1 - \cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$

$- \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{\pi}{10}\right)$ A1

$= -1 - \cos\frac{\pi}{10} - \cos\frac{\pi}{5} - \cos\frac{3\pi}{10} - \cos\frac{4\pi}{10}$ M1

$$= -\begin{pmatrix} 1+\cos\left(-\frac{2\pi}{5}\right)+\cos\left(-\frac{3\pi}{10}\right) \\ +\cos\left(-\frac{\pi}{5}\right)+\cos\left(-\frac{\pi}{10}\right) \end{pmatrix}$$ A1

$$= -\operatorname{Re} S$$

$$\therefore \frac{\operatorname{Re} S}{\operatorname{Im} S} = -1$$ AG

[5]

(d) (i) $\cos\left(-\frac{\pi}{5}\right)$

$$= \cos\frac{\pi}{5}$$

$$= 2\cos^2\frac{\pi}{10} - 1$$ (A1) for substitution

$$= 2\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - 1$$

$$= \frac{10+2\sqrt{5}}{8} - 1$$ (M1) for valid approach

$$= \frac{10+2\sqrt{5}-8}{8}$$

$$= \frac{1+\sqrt{5}}{4}$$ A1

(ii) $\cos\left(-\frac{2\pi}{5}\right)$

$$= 2\cos^2\left(-\frac{\pi}{5}\right) - 1$$ (A1) for substitution

$$= 2\left(\frac{1+\sqrt{5}}{4}\right)^2 - 1$$

$$= \frac{1+2\sqrt{5}+5}{8} - 1$$ (M1) for valid approach

$$= \frac{6+2\sqrt{5}-8}{8}$$

$$= \frac{\sqrt{5}-1}{4}$$ A1

[6]

(e) $\operatorname{Im} S$

$$= -\operatorname{Re} S$$

$$= -\begin{pmatrix} 1 + \cos\left(-\frac{2\pi}{5}\right) + \cos\left(-\frac{3\pi}{10}\right) \\ + \cos\left(-\frac{\pi}{5}\right) + \cos\left(-\frac{\pi}{10}\right) \end{pmatrix}$$ M1

$$= -\left(1 + \frac{\sqrt{5}-1}{4} + \frac{\sqrt{10-2\sqrt{5}}}{4} + \frac{1+\sqrt{5}}{4} + \frac{\sqrt{10+2\sqrt{5}}}{4}\right)$$ A1

$$= -\left(1 + \frac{\sqrt{5}}{2} + \frac{\sqrt{10-2\sqrt{5}} + \sqrt{10+2\sqrt{5}}}{4}\right)$$

$$= -\left(\frac{4}{4} + \frac{2\sqrt{5}}{4} + \frac{\sqrt{10-2\sqrt{5}} + \sqrt{10+2\sqrt{5}}}{4}\right)$$

$$= -\frac{4 + 2\sqrt{5} + \sqrt{10-2\sqrt{5}} + \sqrt{10+2\sqrt{5}}}{4}$$ AG

[2]

12. (a) (i) $f(x)=g(x)$

$\therefore \sin 2\pi y = -\sin \pi y$ M1

$2\sin \pi y \cos \pi y + \sin \pi y = 0$ A1

$\sin \pi y(2\cos \pi y + 1) = 0$ A1

$\sin \pi y = 0$ or $\cos \pi y = -\frac{1}{2}$

$\pi y = 0$ or $\pi y = \frac{2\pi}{3}$ A1

$y = 0$ (*Rejected*) or $y = \frac{2}{3}$

$\therefore r = \frac{2}{3}$ AG

(ii) The area of the region

$= \int_{\frac{2}{3}}^{1} (g(y) - f(y))\mathrm{d}y$ A1

$= \int_{\frac{2}{3}}^{1} (-\sin \pi y - \sin 2\pi y)\mathrm{d}y$

$= \left[\frac{1}{\pi}\cos \pi y + \frac{1}{2\pi}\cos 2\pi y\right]_{\frac{2}{3}}^{1}$ A1

$= \left(\frac{1}{\pi}\cos \pi(1) + \frac{1}{2\pi}\cos 2\pi(1)\right)$
$-\left(\frac{1}{\pi}\cos \pi\left(\frac{2}{3}\right) + \frac{1}{2\pi}\cos 2\pi\left(\frac{2}{3}\right)\right)$ M1

$= \left(-\frac{1}{\pi} + \frac{1}{2\pi}\right) - \left(\frac{1}{\pi}\left(-\frac{1}{2}\right) + \frac{1}{2\pi}\left(-\frac{1}{2}\right)\right)$ A1

$= -\frac{1}{2\pi} - \left(-\frac{1}{2\pi} - \frac{1}{4\pi}\right)$ M1

$= -\frac{1}{2\pi} + \frac{1}{2\pi} + \frac{1}{4\pi}$

$= \frac{1}{4\pi}$ AG

[9]

(b)	$a\sin 2\pi\left(\frac{3}{4}\right)=-\frac{\sqrt{2}}{2}$	(M1) for substitution
	$-a=-\frac{\sqrt{2}}{2}$	A1
	$a=\frac{\sqrt{2}}{2}$	A1

[3]

(c)	$f(x)=g(x)$	
	$\therefore a\sin 2\pi y=-\sin \pi y$	M1
	$2a\sin \pi y\cos \pi y+\sin \pi y=0$	A1
	$\sin \pi y(2a\cos \pi y+1)=0$	A1
	$2a\cos \pi y+1=0$	M1
	$2a\cos \pi y=-1$	
	$\cos \pi y=-\frac{1}{2a}$	A1
	$\therefore \sin \pi y$	
	$=\sqrt{1-\cos^2 \pi y}$	A1
	$=\sqrt{1-\left(-\frac{1}{2a}\right)^2}$	
	$=\sqrt{1-\frac{1}{4a^2}}$	M1
	$=\sqrt{\frac{4a^2-1}{4a^2}}$	A1
	$\pi y=\arcsin\sqrt{\frac{4a^2-1}{4a^2}}$	M1
	$\therefore r=\frac{1}{\pi}\arcsin\sqrt{\frac{4a^2-1}{4a^2}}$	AG

[9]