Chapter 18 Solution

Exercise 77

1. (a) (i) The required probability

$$= \frac{3+2+4+3}{20}$$

(A1) for correct formula

$$= \frac{3}{5}$$

A1 N2

(ii) The required probability

$$= \frac{3+5}{3+3+5}$$

(A1) for correct formula

$$= \frac{8}{11}$$

A1 N2

[4]

(b) The required probability

$$= \left(\frac{3+2+3+3}{20}\right)\left(\frac{3+2+3+3-1}{20-1}\right)$$

A2

$$= \left(\frac{11}{20}\right)\left(\frac{10}{19}\right)$$

$$= \frac{11}{38}$$

A1 N1

[3]

2. (a) (i) The required probability

$$= \frac{2+10+3+5+10}{50}$$ (A1) for correct formula

$$= \frac{3}{5}$$ A1 N2

(ii) The required probability

$$= \frac{3+5+10}{10+3+5+10}$$ (A1) for correct formula

$$= \frac{9}{14}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{5+10}{50}\right)\left(\frac{5+10-1}{50-1}\right)$$ A2

$$= \left(\frac{15}{50}\right)\left(\frac{14}{49}\right)$$

$$= \frac{3}{35}$$ A1 N1

[3]

3. (a) (i) The required probability

$$= \frac{2+1+5+3+4+2+1}{25}$$ (A1) for correct formula

$$= \frac{18}{25}$$ A1 N2

(ii) The required probability

$$= \frac{5}{1+5+2}$$ (A1) for correct formula

$$= \frac{5}{8}$$ A1 N2

[4]

(b) The required probability

$$= \left(\frac{4+2+1}{25}\right)\left(\frac{4+2+1-1}{25-1}\right)$$ A2

$$= \left(\frac{7}{25}\right)\left(\frac{6}{24}\right)$$

$$= \frac{7}{100}$$ A1 N1

[3]

4. (a) $\frac{5+15+a}{100}=\frac{6}{25}$ (M1) for setting equation

$20+a=24$

$a=4$ A1

$5+15+4+\cdots+15+b=100$

$b=6$ A1 N3

[3]

(b) The required probability

$=\frac{15+4+10+10+15+6}{15+4+5+5+10+10+15+6}$ (A1) for correct formula

$=\frac{6}{7}$ A1 N2

[2]

(c) The required probability

$=\left(\frac{6}{100}\right)\left(\frac{6-1}{100-1}\right)$ A2

$=\left(\frac{6}{100}\right)\left(\frac{5}{99}\right)$

$=\frac{1}{330}$ A1 N1

[3]

Exercise 78

1. (a) (i) $a+9=13$
$a=4$

(M1) for valid approach
A1 N2

(ii) $21+4+b=30$
$b=5$

(M1) for valid approach
A1 N2

[4]

(b) The required probability
$=\frac{4}{30}$
$=\frac{2}{15}$

(M1) for valid approach
A1 N2

[2]

2. (a) (i) $17+15-h+10=40$
$h=2$

(M1) for valid approach
A1 N2

(ii) $2+k=15$
$k=13$

(M1) for valid approach
A1 N2

[4]

(b) The required probability
$=\frac{2}{40}$
$=\frac{1}{20}$

(M1) for valid approach
A1 N2

[2]

3. (a) (i) $p=0.4$

A1 N1

(ii) $0.4+q=0.6$
$q=0.2$

(M1) for valid approach
A1 N2

[3]

(b) $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
$0.9=\mathrm{P}(A)+0.6-0.4$
$\mathrm{P}(A)=0.7$

(M1) for valid approach
(A1) for substitution
A1 N2

[3]

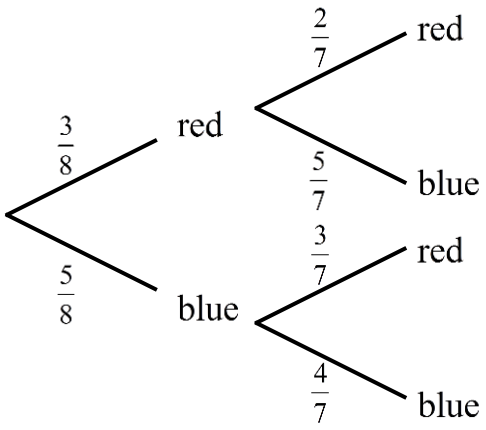
4. (a) (i) $a = 0.3$ A1 N1

(ii) $0.3 + b = 1 - 0.6$ (M1) for valid approach
$b = 0.1$ A1 N2
[3]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach
$1 - 0.3 = 0.6 + 0.2 - \mathrm{P}(A \cap B)$ (A1) for substitution
$\mathrm{P}(A \cap B) = 0.1$ A1 N2
[3]

Exercise 79

1. (a)



A3 N3

[3]

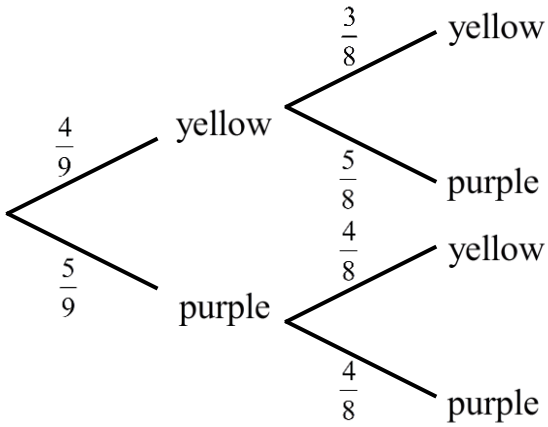
(b) The required probability

$$=\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)+\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$$ (M1)(A1) for correct formula

$$=\frac{15}{28}$$ A1 N2

[3]

2. (a)



A3 N3

[3]

(b) The required probability

$$=\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)+\frac{5}{9}$$ (M1)(A1) for correct formula

$$=\frac{5}{6}$$ A1 N2

[3]

3. (a) $x = \frac{5}{8}$ A1 N1

[1]

(b) $\mathrm{P}(B)$

$= \left(\frac{3}{8}\right)\left(\frac{1}{5}\right) + \left(\frac{5}{8}\right)\left(\frac{2}{5}\right)$ (M1)(A1) for correct formula

$= \frac{13}{40}$ A1 N2

[3]

(c) $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A \mid B) = \frac{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}{\frac{13}{40}}$ (A1) for substitution

$\mathrm{P}(A \mid B) = \frac{3}{13}$ A1 N2

[3]

4. (a) $\mathrm{P}(B \mid A') = \frac{3}{5}$ A1 N1

[1]

(b) $\mathrm{P}(B)$

$= \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{5}\right)$ (M1)(A1) for correct formula

$= \frac{2}{3}$ A1 N2

[3]

(c) $\mathrm{P}(A' \mid B) = \frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (M1) for valid approach

$\mathrm{P}(A' \mid B) = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)}{\frac{2}{3}}$ (A1) for substitution

$\mathrm{P}(A' \mid B) = \frac{3}{5}$ A1 N2

[3]

Exercise 80

1. (a)

$\mathrm{P}(A)$
$= \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$ (M1) for valid approach
$= 0.08 + 0.12$
$= 0.2$ A1 N2

[2]

(b)

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ (M1) for valid approach
$0.08 = 0.2 \times \mathrm{P}(B)$
$\mathrm{P}(B) = 0.4$ (A1) for correct value
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (A1) for correct formula
$\mathrm{P}(A \cup B) = 0.2 + 0.4 - 0.08$
$\mathrm{P}(A \cup B) = 0.52$ A1 N3

[4]

2. (a)

$\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$ (M1) for valid approach
$0.3 = \mathrm{P}(A \cap B) + 0.15$
$\mathrm{P}(A \cap B) = 0.15$ A1 N2

[2]

(b)

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ (M1) for valid approach
$0.15 = \mathrm{P}(A) \times 0.3$
$\mathrm{P}(A) = 0.5$ (A1) for correct value
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (A1) for correct formula
$\mathrm{P}(A \cup B) = 0.5 + 0.3 - 0.15$
$\mathrm{P}(A \cup B) = 0.65$ A1 N3

[4]

3.

$\mathrm{P}(A) = \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$ (M1) for valid approach
$0.4 = \mathrm{P}(A \cap B) + 0.28$
$\mathrm{P}(A \cap B) = 0.12$ A1
$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ (M1) for valid approach
$0.12 = 0.4 \times \mathrm{P}(B)$
$\mathrm{P}(B) = 0.3$ (A1) for correct value
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (A1) for correct formula
$\mathrm{P}(A \cup B) = 0.4 + 0.3 - 0.12$
$\mathrm{P}(A \cup B) = 0.58$ A1 N4

[6]

4. $P(A \cap B) = P(A) \times P(B)$ (M1) for valid approach

$0.21 = 0.7P(B)$

$P(B) = 0.3$ A1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (A1) for correct formula

$P(A \cup B) = 0.7 + 0.3 - 0.21$

$P(A \cup B) = 0.79$ A1

$P(A' \cap B')$

$= 1 - P(A \cup B)$ (M1)(A1) for correct formula

$= 1 - 0.79$

$= 0.21$ A1 N4

[7]

Exercise 81

1. (a) $\mathrm{P}(C \cap D)$
$= \mathrm{P}(C) \times \mathrm{P}(D)$
$= 2k^2 \times 3k^2$ (A1) for substitution
$= 6k^4$ A1 N2
[2]

(b) $6k^4 = 0.0096$ (A1) for correct equation
$k^4 = 0.0016$
$k = 0.2$ A1 N2
[2]

(c) $\mathrm{P}(C) = \mathrm{P}(C \cap D) + \mathrm{P}(C \cap D')$
$2(0.2)^2 = 6(0.2)^4 + \mathrm{P}(C \cap D')$ (A1) for substitution
$\mathrm{P}(C \cap D') = 0.0704$
$\mathrm{P}(D' \mid C)$
$= \dfrac{\mathrm{P}(D' \cap C)}{\mathrm{P}(C)}$
$= \dfrac{0.0704}{2(0.2)^2}$ (A1) for substitution
$= 0.88$ A1 N2
[3]

2. (a) $\mathrm{P}(E \cap F)$
$= \mathrm{P}(E) \times \mathrm{P}(F)$
$= 4k^3 \times k$ (A1) for substitution
$= 4k^4$ A1 N2
[2]

(b) $4k^4 = \dfrac{1}{2500}$ (A1) for correct equation
$k^4 = \dfrac{1}{10000}$
$k = \dfrac{1}{10}$ A1 N2
[2]

(c) $\mathrm{P}(E \cup F)$
$= \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F)$
$= 4\left(\dfrac{1}{10}\right)^3 + \dfrac{1}{10} - 4\left(\dfrac{1}{10}\right)^4$ (A1) for substitution
$= \dfrac{259}{2500}$ A1 N2
[2]

3. (a) $\mathrm{P}(A \cap B)$

$= \mathrm{P}(A) \times \mathrm{P}(B)$

$= 2k \times 1.5(2k)$ (A1) for substitution

$= 6k^2$ A1

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$6k - 1 = 2k + 1.5(2k) - 6k^2$ (A1) for correct equation

$6k - 1 = 5k - 6k^2$

$6k^2 + k - 1 = 0$ (M1) for valid approach

$(3k - 1)(2k + 1) = 0$

$k = \frac{1}{3}$ or $k = -\frac{1}{2}$ (*Rejected*) A1 N3

[5]

(b) $\mathrm{P}(B \mid A)$

$= \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}$

$= \frac{6\left(\frac{1}{3}\right)^2}{2\left(\frac{1}{3}\right)}$ (A1) for substitution

$= 1$ A1 N2

[2]

4. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ R1

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times 3\mathrm{P}(A)$

$\mathrm{P}(A \cap B) = 3\mathrm{P}(A)^2$ (A1) for correct formula

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach

$0.93 = \mathrm{P}(A) + 3\mathrm{P}(A) - 3\mathrm{P}(A)^2$ A1

$3\mathrm{P}(A)^2 - 4\mathrm{P}(A) + 0.93 = 0$

$300\mathrm{P}(A)^2 - 400\mathrm{P}(A) + 93 = 0$

$(30\mathrm{P}(A) - 31)(10\mathrm{P}(A) - 3) = 0$

$\mathrm{P}(A) = \frac{31}{30}$ (*Rejected*) or $\mathrm{P}(A) = \frac{3}{10}$ A2

$\therefore \mathrm{P}(B)$

$= 3\left(\frac{3}{10}\right)$

$= \frac{9}{10}$ A1 N6

[7]

Exercise 82

1. (a) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach

$1 = 0.4 + 0.65 - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = 0.05$ A1 N2

[2]

(b) $\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B) = \mathrm{P}(B)$ (M1) for valid approach

$0.05 + \mathrm{P}(A' \cap B) = 0.65$

$\mathrm{P}(A' \cap B) = 0.6$ A1 N2

[2]

(c) (i) $\mathrm{P}(A \cap C)$

$= \mathrm{P}(A \mid C) \times \mathrm{P}(C)$ (M1) for valid approach

$= 0.78 \times 0.7$ (A1) for substitution

$= 0.546$ A1 N3

(ii) $\mathrm{P}(A \cap C)$

$= 0.546$

$\neq 0$ R1

Thus, A and C are **not** mutually exclusive. AG N0

Valid reasoning

(iii) $\mathrm{P}(A) \times \mathrm{P}(C)$

$= 0.4 \times 0.7$

$= 0.28$ A1

$\neq 0.546$ R1

$= \mathrm{P}(A \cap C)$

Thus, A and C are **not** independent. AG N0

[6]

(d) $\mathrm{P}(A \cap C) + \mathrm{P}(A' \cap C) = \mathrm{P}(C)$ (M1) for valid approach

$0.546 + \mathrm{P}(A' \cap C) = 0.7$ (A1) for substitution

$\mathrm{P}(A' \cap C) = 0.154$ (A1) for correct value

$\mathrm{P}(A) + \mathrm{P}(A') = 1$

$0.4 + \mathrm{P}(A') = 1$

$\mathrm{P}(A') = 0.6$ A1

$\mathrm{P}(C \mid A')$

$= \dfrac{\mathrm{P}(C \cap A')}{\mathrm{P}(A')}$ (M1) for valid approach

$= \dfrac{0.154}{0.6}$

$= 0.256666666$

$= 0.257$ A1 N3

[6]

2. (a) $\mathrm{P}(A \cup T) = \mathrm{P}(A) + \mathrm{P}(T) - \mathrm{P}(A \cap T)$ (M1) for valid approach

$1 = 0.55 + 0.7 - \mathrm{P}(A \cap T)$

$\mathrm{P}(A \cap T) = 0.25$

Thus, the required percentage is 25%. A1 N2

[2]

(b) $\mathrm{P}(A \cup T) - \mathrm{P}(A \cap T)$ (M1) for valid approach

$= 1 - 0.25$

$= 0.75$

Thus, the required percentage is 75%. A1 N2

[2]

(c) (i) $\mathrm{P}(M \cap A)$

$= \mathrm{P}(A \mid M) \times \mathrm{P}(M)$ (M1) for valid approach

$= 0.72 \times 0.63$ (A1) for substitution

$= 0.4536$ A1 N3

(ii) $\mathrm{P}(M) \times \mathrm{P}(A)$

$= 0.63 \times 0.55$

$= 0.3465$ A1

$\neq 0.4536$ R1

$= \mathrm{P}(M \cap A)$

Thus, M and A are **not** independent. AG N0

[5]

(d) $\mathrm{P}(M \cap A) + \mathrm{P}(M' \cap A) = \mathrm{P}(A)$ (M1) for valid approach

$0.4536 + \mathrm{P}(M' \cap A) = 0.55$ (A1) for substitution

$\mathrm{P}(M' \cap A) = 0.0964$ (A1) for correct value

$\mathrm{P}(M) + \mathrm{P}(M') = 1$

$0.63 + \mathrm{P}(M') = 1$

$\mathrm{P}(M') = 0.37$ A1

$\mathrm{P}(A \mid M')$

$= \dfrac{\mathrm{P}(A \cap M')}{\mathrm{P}(M')}$ (M1) for valid approach

$= \dfrac{0.0964}{0.37}$

$= 0.26054054$

$= 0.261$ A1 N3

[6]

3. (a) $\mathrm{P}(F \cup R) = \mathrm{P}(F) + \mathrm{P}(R) - \mathrm{P}(F \cap R)$ (M1) for valid approach

$1 = 0.85 + 0.45 - \mathrm{P}(F \cap R)$

$\mathrm{P}(F \cap R) = 0.3$

Thus, the required percentage is 30%. A1 N2

[2]

(b) $\mathrm{P}(F \cup R) - \mathrm{P}(F \cap R)$ (M1) for valid approach

$= 1 - 0.3$

$= 0.7$

Thus, the required percentage is 70%. A1 N2

[2]

(c) (i) $\mathrm{P}(R \mid F)$

$= \dfrac{\mathrm{P}(R \cap F)}{\mathrm{P}(F)}$

$= \dfrac{0.3}{0.85}$ (M1) for substitution

$= 0.352941176$

$= 0.353$ A1 N2

(ii) $\mathrm{P}(R \mid (F \cap R)')$

$= \dfrac{\mathrm{P}(R \cap (F \cap R)')}{\mathrm{P}((F \cap R)')}$

$= \dfrac{\mathrm{P}(F' \cap R)}{1 - \mathrm{P}(F \cap R)}$

$= \dfrac{1 - 0.85}{1 - 0.3}$ (M1) for substitution

$= 0.214285714$

$= 0.214$ A1 N2

[4]

(d) (i) $\mathrm{P}(F \cap T)$

$= \mathrm{P}(F \mid T) \times \mathrm{P}(T)$ (M1) for valid approach

$= 0.9 \times 0.6$

$= 0.54$ A1

$\mathrm{P}(F \cap T) \neq 0$ R1

Thus, F and T are **not** mutually exclusive. AG N0

(ii) $\mathrm{P}(F) \times \mathrm{P}(T)$

$= 0.85 \times 0.6$

$= 0.51$ A1

$\neq 0.54$ R1

Thus, F and T are **not** independent. AG N0

(iii) $\mathrm{P}(F \cap T) + \mathrm{P}(F \cap T') = \mathrm{P}(F)$

$0.54 + \mathrm{P}(F \cap T') = 0.85$ (M1) for substitution

$\mathrm{P}(F \cap T') = 0.31$

Thus, the required percentage is 31%. A1 N2

[7]

4. (a) $\mathrm{P}(Q \cup T) = \mathrm{P}(Q) + \mathrm{P}(T) - \mathrm{P}(Q \cap T)$ (M1) for valid approach

$1 - 0.25 = 0.35 + 0.5 - \mathrm{P}(Q \cap T)$

$\mathrm{P}(Q \cap T) = 0.1$

Thus, the required percentage is 10%. A1 N2

[2]

(b) $\mathrm{P}(Q \cap T) + \mathrm{P}(Q' \cap T) = \mathrm{P}(T)$ (M1) for valid approach

$0.1 + \mathrm{P}(Q' \cap T) = 0.5$

$\mathrm{P}(Q' \cap T) = 0.4$

Thus, the required percentage is 40%. A1 N2

[2]

(c) (i) $\mathrm{P}(Q' \cap T \mid Q \cup T)$

$= \dfrac{\mathrm{P}(Q' \cap T)}{\mathrm{P}(Q \cup T)}$

$= \dfrac{0.4}{1 - 0.25}$ (M1) for substitution

$= 0.533333333$

$= 0.533$ A1 N2

(ii) $\mathrm{P}(Q \mid T)$

$= \dfrac{\mathrm{P}(Q \cap T)}{\mathrm{P}(T)}$

$= \dfrac{0.1}{0.5}$ (M1) for substitution

$= 0.2$ A1 N2

[4]

(d) (i) $\mathrm{P}(T \cap G)$

$= \mathrm{P}(T \mid G) \times \mathrm{P}(G)$ (M1) for valid approach

$= 0.95 \times 0.4$

$= 0.38$ A1

$\mathrm{P}(T \cap G) \neq 0$ R1

Thus, T and G are **not** mutually exclusive. AG N0

(ii) $\mathrm{P}(T) \times \mathrm{P}(G)$

$= 0.5 \times 0.4$

$= 0.2$ A1

$\neq 0.38$ R1

$= \mathrm{P}(T \cap G)$

Thus, T and G are **not** independent. AG N0

(iii) $\mathrm{P}(T \cap G) + \mathrm{P}(T \cap G') = \mathrm{P}(T)$

$0.38 + \mathrm{P}(T \cap G') = 0.5$ (M1) for substitution

$\mathrm{P}(T \cap G') = 0.12$

Thus, the required percentage is 12%. A1 N2

[7]