HKDSE Mathematics Compulsory Part 2020 Mock Exam Set 1 Paper 1 Marking Scheme

Section A(1)

Solution	Marks	Remarks
1. $\frac{2+x}{3}=4y-\frac{1-2x}{4}$		
$4(2+x)=48y-3(1-2x)$	1M	
$8+4x=48y-3+6x$		
$11-48y=2x$	1M	for putting x on one side
$x=\frac{11-48y}{2}$	1A	or equivalent
	------(3)	
2. $\frac{(-m^2n^4)^3}{m^{10}n^{20}}$		
$=\frac{-m^6n^{12}}{m^{10}n^{20}}$	1M	for $(a^m)^n=a^{mn}$
$=-m^{6-10}n^{12-20}$	1M	for $\frac{a^m}{a^n}=a^{m-n}$
$=-m^{-4}n^{-8}$		
$=-\frac{1}{m^4n^8}$	1A	
	------(3)	
3. (a) $8xy^2-4y^3-4y^2$		
$=4y^2(2x-y-1)$	1A	or equivalent
(b) $8xy^2-4y^3-4y^2-2x+y+1$		
$=4y^2(2x-y-1)-(2x-y-1)$	1M	for using the result of (a)
$=(2x-y-1)(4y^2-1)$	1M	
$=(2x-y-1)(2y+1)(2y-1)$	1A	or equivalent
	------(4)	

Solution	Marks	Remarks
4. L.H.S.		
$= a(x-h)^2 + k$		
$= a(x^2 - 2hx + h^2) + k$		
$= ax^2 - 2ahx + ah^2 + k$	1M	for expanding any side
R.H.S.		
$= k(1 - 3ax + 2x^2) + ah^2$		
$= k - 3akx + 2kx^2 + ah^2$		
$= 2kx^2 - 3akx + ah^2 + k$		
$\therefore a = 2k$ and $2ah = 3ak$	1M	
$h = \frac{3}{2}k$		
$a : h : k$		
$= 2k : \frac{3}{2}k : k$	1M	
$= 2 : \frac{3}{2} : 1$		
$= 4 : 3 : 2$	1A	
	------(4)	
5. (a) Let $\$S$ be the selling price.		
Then the cost is $\$(S - 4560)$.		
$(S - 4560)(1 + 120\%)(1 - 20\%) = S$	1M	
$1.76(S - 4560) = S$		
$1.76S - 8025.6 = S$		
$0.76S = 8025.6$		
$S = 10560$	1A	
Thus, the selling price is \$10560.		
(b) The cost		
$= 10560 - 4560$	1M	
$= \$6000$		
The percentage profit		
$= \frac{10560 - 6000}{6000} \times 100\%$		
$= 76\%$		
Hence, the claim is disagreed.	1A	f.t.
	------(4)	

Solution			Marks	Remarks
6.	(a)	$1-\frac{x}{2}\le\frac{x}{3}$ or $\frac{3x+15}{2}>30-x$		
		$6-3x\le 2x$ or $3x+15>60-2x$		
		$6\le 5x$ or $5x>45$		
		$x\ge\frac{6}{5}$ or $x>9$	1A+1A	
		$\therefore x\ge\frac{6}{5}$	1A	
	(b)	4	1A	
			------(4)	
7.	(a)	$\because AC$ is a diameter		
		$\therefore \angle ABC=90^\circ$		
		$(x-25)^\circ+(y+45)^\circ=90^\circ$		
		$x+y=70$		
		$y=70-x$	1A	
	(b)	$\angle BAE=180^\circ-110^\circ-(x-25)^\circ$		
		$\angle BAE=(95-x)^\circ$		
		$\angle BDC=180^\circ-80^\circ-(y+45)^\circ$		
		$\angle BDC=(55-y)^\circ$		
		$\therefore 95-x=55-y$	1A	
		$95-x=55-(70-x)$		
		$x=55$	1A	
		$y=15$	1A	
			------(4)	
8.	(a)	The polar coordinates of A' are $(10, 315^\circ)$.	1A	
		The polar coordinates of B' are $(24, 225^\circ)$.	1A	
	(b)	Note that $\angle A'OB'=90^\circ$.		
		Area of the triangle $OA'B'$		
		$=\frac{(10)(24)}{2}$	1M	
		$=120$	1A	
			------(4)	

Solution				Marks	Remarks
9.	(a)	(i)	The required total frequency		
			$=(a^2+4a-2)-(3a+8)$	1M	
			$=a^2+a-10$	1A	
		(ii)	$\dfrac{a^2+a-10}{10a}=\dfrac{1}{4}$	1M	
			$4a^2+4a-40=10a$		
			$4a^2-6a-40=0$	1M	
			$2a^2-3a-20=0$		
			$(2a+5)(a-4)=0$		
			$2a+5=0$ or $a-4=0$		
			$a=-\dfrac{5}{2}$ (*Rejected*) or $a=4$	1A	
				------(5)	

		Solution	Marks	Remarks
10.	(a)	Let H cm and h cm be the heights of the larger pyramid and the smaller pyramid respectively.		
		$2\left(\frac{1}{3}R^2H\right)=81\left(\frac{1}{3}r^2h\right)$	1M	
		$2R^2H=81r^2h$		
		$\frac{R^2}{r^2}=\frac{81h}{2H}$		
		$\frac{R^2}{r^2}=\frac{81h}{2(4.5h)}$	1M	
		$\frac{R^2}{r^2}=\frac{9}{1}$		
		$\frac{R}{r}=\frac{3}{1}$		
		$\therefore R:r=3:1$	1A	
			------(3)	
	(b)	$9:1$	1A	
			------(1)	
	(c)	$R:r=3:1$		
		But $H:h=4.5:1\neq 3:1$	1M	for comparing 2 ratios
		Therefore, the two pyramids are not similar.		
		Thus, the claim is disagreed.	1A	f.t.
			------(2)	

		Solution	Marks	Remarks
11.	(a)	The required average speed		
		$=\dfrac{100\text{ km}}{80\text{ min}}$	1M	
		$=\dfrac{(100\times 1000)\text{ m}}{(80\times 60)\text{ sec}}$		
		$=\dfrac{125}{6}\text{ ms}^{-1}$	1A	r.t. 20.8 ms^{-1}
			------(2)	
	(b)	The required time		
		$=\dfrac{x\text{ km}}{\dfrac{100\text{ km}}{80\text{ min}}}$	1M	
		$=\dfrac{4x}{5}$ minutes after 1:00 pm	1A	
			------(2)	
	(c)	$\dfrac{4x}{5}=40$	1M	
		$4x=200$		
		$x=50$	1A	
			------(2)	

		Solution	Marks	Remarks
12.	(a)	$\angle BAC = \angle DBC$ (*Given*)		
		$\angle ACB = \angle BCD$ (*Common* $\angle$)		
		$\angle ABC = \angle BDC$ ($\angle$ *sum of* Δ)		
		$\therefore \Delta ACB \sim \Delta BCD$ (*A.A.A.*)		
		Any correct proof with correct reasons	2	
		Any correct proof without reasons	1	
			------(2)	
	(b)	$\Delta ACB \sim \Delta BCD$		
		$\therefore \frac{DC}{BC} = \frac{BC}{AC}$		
		$\frac{(70-k)-(k+12)}{k+10} = \frac{k+10}{70-k}$	1M	for correct substitution
		$\frac{58-2k}{k+10} = \frac{k+10}{70-k}$		
		$(58-2k)(70-k) = (k+10)(k+10)$		
		$4060 - 198k + 2k^2 = k^2 + 20k + 100$		
		$k^2 - 218k + 3960 = 0$	1	
		$(k-198)(k-20) = 0$		
		$k = 198$ (*Rejected*) or $k = 20$	1A	
			------(3)	
	(c)	$BD^2 + DC^2$		
		$= 24^2 + (58 - 2(20))^2$	1M	
		$= 900$		
		BC^2		
		$= (20+10)^2$		
		$= 900$		
		$\because BD^2 + DC^2 = BC^2$		
		$\therefore \Delta BCD$ a right-angled triangle.	1A	
			------(2)	

Solution			Marks	Remarks
13.	(a)	The mean		
		$=160$	1A	
		The median		
		$=\dfrac{159+160}{2}$		
		$=159.5$	1A	
		The range		
		$=188-142$		
		$=46$	1A	
		The standard deviation		
		$=12.5$	1A	
			------(4)	
	(b)	Let h cm, $(h+1)$ cm, $(h+2)$ cm and $(h+3)$ cm be the heights of the 4 new students.	1M	
		Note that $(h+3)\text{ cm}\le 188\text{ cm}$.		
		$\dfrac{(160)(20)+h+(h+1)+(h+2)+(h+3)}{20+4}=160+1.25$	1M	
		$\dfrac{4h+3206}{24}=161.25$		
		$4h+3206=3870$		
		$4h=664$		
		$h=166$	1A	
		Thus, the height of these four students are 166 cm, 167 cm, 168 cm and 169 cm.	1A	for all correct
			------(4)	

Solution			Marks	Remarks
14.	(a)	Let $W = k_1x + k_2x^2$, where k_1 and k_2 are non-zero constants.	1A	
		$\begin{cases} 130 = 10k_1 + 100k_2 \\ 10 = -5k_1 + 25k_2 \end{cases}$	1M	
		Solving, we have $k_1 = 3$ and $k_2 = 1$.	1A	
		Therefore, we have $W = 3x + x^2$.		
			------(3)	
	(b)	$\text{p}(x)$		
		$= (x-3)(3x + x^2 - 19) + (-9)$	1M	
		$= x^3 - 28x + 57 - 9$		
		$= x^3 - 28x + 48$	1A	
			------(2)	
	(c)	$\text{p}(2)$		
		$= 2^3 - 28(2) + 48$	1M	for finding a factor
		$= 0$		
		$\therefore (x-2)$ is a factor of $\text{p}(x)$.		
		By long division, $\text{p}(x) = (x-2)(x^2 + 2x - 24)$.	1M	
		$\text{p}(x) = 0$		
		$(x-2)(x^2 + 2x - 24) = 0$		
		$(x-2)(x-4)(x+6) = 0$		
		$x - 2 = 0$, $x - 4 = 0$ or $x + 6 = 0$		
		$x = 2$, 4 or -6		
		Therefore, not all the roots of the equation $\text{p}(x) = 0$ are positive integers.		
		Thus, the claim is disagreed.	1A	f.t.
			------(3)	

		Solution	Marks	Remarks
15.	(a)	Let $\bar{x}$ and σ be the mean and the standard deviation of the examination scores respectively.		
		$-1=\frac{72-\bar{x}}{\sigma}$ and $3=\frac{92-\bar{x}}{\sigma}$	1M	
		Solving, we have $\bar{x}=77$.	1A	
			------(2)	
	(b)	The examination score corresponding to the standard score 1		
		$=72+2(77-72)$	1M	
		$=82$ marks		
		Consider the case that more than half of the examination scores are greater than 82 marks.		
		In this case, the median must be greater than 82 marks, with its standard score greater than 1.		
		Therefore, the standard score of the median of the examination scores may not lie between -1 and 1.		
		Thus, the claim is disagreed.	1A	f.t.
			------(2)	

		Solution	Marks	Remarks
16.	(a)	$a_{29} - a_{17} = 24$		
		$\left[a_1 + (29-1)d\right] - \left[a_1 + (17-1)d\right] = 24$	1M	
		$12d = 24$		
		$d = 2$		
		$a_5 = 2a_2$		
		$a_1 + (5-1)(2) = 2\left[a_1 + (2-1)(2)\right]$		
		$a_1 + 8 = 2a_1 + 4$		
		$a_1 = 4$		
		$\therefore a_n$		
		$= 4 + (n-1)(2)$		
		$= 2n + 2$	1A	
			------(2)	
	(b)	$b_{n+1} - b_n$		
		$= (a_{n+1} + a_{n+2}) - (a_n + a_{n+1})$	1M	
		$= a_{n+2} - a_n$		
		$= \left[2(n+2) + 2\right] - (2n+2)$		
		$= 4$		
		Therefore, the difference between each consecutive term of b_n is 4.		
		Thus, b_n is an arithmetic sequence.	1A	f.t.
			------(2)	
	(c)	S_n		
		$= b_1 + b_2 + \cdots + b_n$		
		$= (a_1 + a_2) + (a_2 + a_3) + \cdots + (a_n + a_{n+1})$		
		$= a_1 + 2(a_2 + \cdots + a_n) + a_{n+1}$		
		$= 2(a_1 + a_2 + \cdots + a_n) + a_{n+1} - a_1$	1M	
		$\therefore a_1 + a_2 + \cdots + a_n$		
		$= \dfrac{S_n + a_1 - a_{n+1}}{2}$		
		$= \dfrac{S_n + 4 - \left[2(n+1) + 2\right]}{2}$		
		$= \dfrac{S_n - 2n}{2}$	1A	
			------(2)	

Solution			Marks	Remarks
17.	(a)	$\begin{cases} x=3 \\ x+3y=60 \end{cases}$	1M	for any one system
		Solving, we have $x=3$ and $y=19$.		
		$\begin{cases} x+3y=60 \\ 3x+y=60 \end{cases}$		
		Solving, we have $x=15$ and $y=15$.		
		$\begin{cases} 3x+y=60 \\ y=0 \end{cases}$		
		Solving, we have $x=20$ and $y=0$.		
		$\begin{cases} x=3 \\ y=0 \end{cases}$		
		Solving, we have $x=3$ and $y=0$.		
		$\begin{cases} x=3 \\ 3x+y=60 \end{cases}$		
		Solving, we have $x=3$ and $y=51$ (*Rejected*).		
		$\begin{cases} x+3y=60 \\ y=0 \end{cases}$	1M	for all systems
		Solving, we have $x=60$ and $y=0$ (*Rejected*).		
		From the above results, we have $a=3$, $b=15$ and $c=0$.	1A+1A	1A for one correct + 1A for all correct
			------(4)	
	(b)	At $(3,19)$: $6y-7-8x=83$		
		At $(15,15)$: $6y-7-8x=-37$		
		At $(20,0)$: $6y-7-8x=-167$		
		At $(3,0)$: $6y-7-8x=-31$		
		The required greatest value is 83.	1A	
		The required least value is -167.	1A	
			------(2)	

Solution			Marks	Remarks
18.	(a)	G		
		$= (12, 6)$	1A	
			------(1)	
	(b)	$4x + 3y - 4k = 0$		
		$y = \frac{4k - 4x}{3}$		
		$\therefore x^2 + \left(\frac{4k - 4x}{3}\right)^2 - 24x - 12\left(\frac{4k - 4x}{3}\right) + 80 = 0$	1M	
		$x^2 + \frac{16k^2 - 32kx + 16x^2}{9} - 24x - 4(4k - 4x) + 80 = 0$		
		$9x^2 + 16k^2 - 32kx + 16x^2 - 216x$		
		$-36(4k - 4x) + 720 = 0$		
		$9x^2 + 16k^2 - 32kx + 16x^2 - 216x$		
		$-144k + 144x + 720 = 0$		
		$25x^2 + (-32k - 72)x + (16k^2 - 144k + 720) = 0$		
		As there is at least one point of intersection between L and C, $\Delta \geq 0$.		
		$(-32k - 72)^2 - 4(25)(16k^2 - 144k + 720) \geq 0$	1M	
		$1024k^2 + 4608k + 5184 - 100(16k^2 - 144k + 720) \geq 0$		
		$-576k^2 + 19008k - 66816 \geq 0$		
		$k^2 - 33k + 116 \leq 0$		
		$(k - 4)(k - 29) \leq 0$		
		$4 \leq k \leq 29$	1A	
			------(3)	

Solution	Marks	Remarks
(c) For one intersection, $\Delta = 0$.		
$k = 4$ or $k = 29$		
$k = 4$:		
$25x^2 - 200x + 400 = 0$	1M	For either one
$x = 4$		
$y = \dfrac{4(4) - 4(4)}{3}$		
$y = 0$ (*Rejected*)		
$k = 29$:		
$25x^2 - 1000x + 10000 = 0$		
$x = 20$		
$y = \dfrac{4(29) - 4(20)}{3}$		
$y = 12$		
$\therefore H : (20, 12)$	1A	
As $A : (4, 0)$ and $B : (20, 0)$, ΔABH is a right-angled triangle, where $AB \perp BH$.		
Therefore, the orthocentre of ΔABH is B.		
Thus, the required coordinates are $(20, 0)$.	1A	
	------(3)	

Solution			Marks	Remarks
19.	(a)	By cosine formula,		
		$\cos\angle ACB = \dfrac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$	1M	
		$\cos\angle ACB = \dfrac{15^2 + 13^2 - 16^2}{2(15)(13)}$		
		$\cos\angle ACB = \dfrac{23}{65}$		
		$\angle ACB \approx 69.27725562°$		
		$\angle ACB \approx 69.3°$	1A	r.t. 69.3°
			------(2)	
	(b)	$\angle DCB \approx 180° - 69.27725562°$		
		$\angle DCB \approx 110.7227444°$		
		By sine formula,		
		$\dfrac{\sin\angle CDB}{BC} = \dfrac{\sin\angle DCB}{BD}$	1M	
		$\dfrac{\sin\angle CDB}{13} \approx \dfrac{\sin 110.7227444°}{22}$		
		$\sin\angle CDB \approx \dfrac{13\sin 110.7227444°}{22}$		
		$\angle CDB \approx 33.55102753°$		
		The area of the triangle CDB		
		$= \dfrac{1}{2}(BC)(BD)\sin\angle DBC$	1M	
		$\approx \dfrac{1}{2}(13)(22)\sin(69.27725562° - 33.55102753°)$		
		$\approx 83.49954396 \text{ m}^2$		
		$\approx 83.5 \text{ m}^2$	1A	r.t. 83.5 m^2
			------(3)	

Solution	Marks	Remarks
(c) (i) The two triangular pyramids $EACB$ and $ECDB$ share the same height with respect to the bases ACB and CDB.	1M	
Therefore, the ratio of the volume of $EACB$ to the volume of $ECDB$ is equal to the ratio of the area of ΔACB to the area of ΔCDB.	1M	
The area of the triangle ACB		
$=\frac{1}{2}(AC)(BC)\sin\angle ACB$	1M	
$\approx\frac{1}{2}(15)(13)\sin 69.27725562^\circ$		
$\approx 91.19210492\ \text{m}^2$		
The required ratio		
$\approx 91.19210492\ \text{m}^2 : 83.49954396\ \text{m}^2$		
$\approx 1:0.916$	1A	r.t. 0.916
(ii) Let h m be the perpendicular distance from E to the straight line AB.		
Volume of $EABD=\frac{1}{3}(\text{Area of }\Delta ABD)(h)$	1M	
$1170=\frac{1}{3}(91.19210492+83.49954396)(h)$	1A	
$h\approx 20.09254605\ \text{m}$		
$h\approx 20.1\ \text{m}$	1A	r.t. $20.1\ \text{m}$
Thus, the required distance is 20.1 m.		
	------(7)	