

Chapter 14 Solution

Exercise 48

1. (a) $\pi r^2 = 9\pi$ (M1) for setting equation
 $r^2 = 9$
 $r = 3 \text{ cm}$ A1 N2 [2]
- (b) $12\pi \text{ cm}^3$ A1 N1 [1]
- (c) The slant height l of the circular cone
 $= \sqrt{3^2 + 4^2}$ (M1) for valid approach
 $= 5$
The total surface area
 $= \pi r^2 + \pi r l$ (M1) for valid approach
 $= \pi(3)^2 + \pi(3)(5)$ (A1) for substitution
 $= 75.39822369$
 $= 75.4 \text{ cm}^2$ A1 N4 [4]
2. (a) $\pi r^2 = 37$ (M1) for setting equation
 $r = \sqrt{\frac{37}{\pi}}$
 $r = 3.431831259$
 $r = 3.43 \text{ cm}$ A1 N2 [2]
- (b) 84.7 cm^3 A2 N2 [2]
- (c) The total surface area
 $= 2\pi r^2 + \pi r^2$ (M1) for valid approach
 $= 3\pi r^2$
 $= 3(37)$ (A1) for substitution
 $= 111 \text{ cm}^2$ A1 N3 [3]

3. (a) $V = \frac{1}{3}\pi r^2 h$ (M1) for setting equation
 $150 = \frac{1}{3}\pi r^2 (13)$
 $r = \sqrt{\frac{450}{13\pi}}$
 $r = 3.319400418$
 $r = 3.32 \text{ cm}$ A1 N2 [2]
- (b) $l = \sqrt{3.319400418^2 + 13^2}$ (M1) for valid approach
 $l = 13.41709429$
 $l = 13.4 \text{ cm}$ A1 N2 [2]
- (c) The curved surface area (M1) for valid approach
 $= \pi r l$
 $= \pi(3.319400418)(13.41709429)$
 $= 139.9161959$
 $= 140 \text{ cm}^2$ A1 N2 [2]
4. (a) $A = \pi r l$ (M1) for setting equation
 $369\pi = \pi r(41)$
 $r = 9 \text{ cm}$ A1 N2 [2]
- (b) The vertical height h (M1) for valid approach
 $= \sqrt{41^2 - 9^2}$
 $= 40 \text{ cm}$ A1 N2 [2]
- (c) The volume (M1) for valid approach
 $= \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(9)^2(40)$
 $= 1080\pi \text{ cm}^3$ A1 N2 [2]

Exercise 49

1. (a) The volume
$$= \frac{1}{3}\pi R^2 H + \pi r^2 h$$
 (M2) for valid approach
$$= \frac{1}{3}\pi(12)^2(16) + \pi(12)^2(5)$$
 (A1) for substitution
$$= 4674.689869$$

$$= 4670 \text{ m}^3$$
 A1 N4 [4]
- (b) The slant height of the top
$$= \sqrt{12^2 + 16^2}$$
 (M1) for valid approach
$$= 20$$

The area
$$= \pi r l$$
 (M1) for valid approach
$$= \pi(12)(20)$$

$$= 753.9822369$$

$$= 754 \text{ m}^2$$
 A1 N3 [3]
2. (a) The volume
$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
 (M2) for valid approach
$$= \frac{1}{3}\pi(8)^2(6) + \frac{2}{3}\pi(8)^3$$
 (A1) for substitution
$$= 1474.454152$$

$$= 1470 \text{ cm}^3$$
 A1 N4 [4]
- (b) The slant height of the circular cone
$$= \sqrt{6^2 + 8^2}$$
 (M1) for valid approach
$$= 10$$

The total surface area
$$= \pi r l + 2\pi r^2$$
 (M1) for valid approach
$$= \pi(8)(10) + 2\pi(8)^2$$

$$= 653.4512719$$

$$= 653 \text{ cm}^2$$
 A1 N3 [3]

3. (a) $V = \frac{2}{3}\pi r^3 + \pi r^2 h$ (M2) for valid approach
- $54000\pi = \frac{2}{3}\pi r^3 + \pi r^2(40)$ (A1) for substitution
- $54000 = \frac{2}{3}r^3 + 40r^2$
- $\frac{2}{3}r^3 + 40r^2 - 54000 = 0$ (M1) for setting equation
- $r = 30 \text{ m}$ A1 N5 [5]
- (b) The area
- $= 2\pi r^2$ (M1) for valid approach
- $= 2\pi(30)^2$
- $= 5654.866776$
- $= 5650 \text{ m}^2$ A1 N2 [2]
4. (a) $A = 2\pi r^2 + 2\pi rh + 2\pi r^2$ (M2) for setting equation
- $28\pi = 4\pi r^2 + 2\pi r(3)$ (A1) for substitution
- $28 = 4r^2 + 6r$
- $2r^2 + 3r - 14 = 0$ (M1) for quadratic equation
- $(2r + 7)(r - 2) = 0$
- $2r + 7 = 0$ or $r - 2 = 0$
- $r = -\frac{7}{2}$ (*Rejected*) or $r = 2 \text{ mm}$ A1 N5 [5]
- (b) The volume
- $= \frac{4}{3}\pi r^3 + \pi r^2 h$ (M1) for valid approach
- $= \frac{4}{3}\pi(2)^3 + \pi(2)^2(3)$
- $= 71.20943348$
- $= 71.2 \text{ mm}^3$ A1 N2 [2]

Exercise 50

1. (a) The volume

$$= \frac{2}{3} \pi r^3$$
 (M1) for valid approach

$$= \frac{2}{3} \pi (22)^3$$

$$= 22301.11905$$
 (A1) for correct value

$$= 22300$$

$$= 2.23 \times 10^4 \text{ cm}^3$$
 A1 N3 [3]
- (b) $V = \pi r^2 h$ (M1) for setting equation
 $22301.11905 = \pi r^2 (26)$ (A1) for substitution
 $r^2 = 273.025641$
 $r = 16.52348756$
 $r = 16.5 \text{ cm}$ A1 N3 [3]
2. (a) The volume

$$= \frac{1}{3} Ah$$
 (M1) for valid approach

$$= \frac{1}{3} (8\pi)^2 (35)$$

$$= 7369.304619$$
 (A1) for correct value

$$= 7370$$

$$= 7.37 \times 10^3 \text{ cm}^3$$
 A1 N3 [3]
- (b) $V = \frac{4}{3} \pi r^3$ (M1) for setting equation
 $7369.304619 = \frac{4}{3} \pi r^3$ (A1) for substitution
 $r^3 = 1759.291886$
 $r = 12.07200203$
 $r = 12.1 \text{ cm}$ A1 N3 [3]

3. (a) The volume
 $= \pi r^2 h$ (M1) for valid approach
 $= \pi(7)^2(100)$
 $= 4900\pi \text{ cm}^3$ A1 N2 [2]
- (b) $V = 10\left(\frac{2}{3}\pi r^3\right)$ (M1) for setting equation
 $4900\pi = \frac{20}{3}\pi r^3$ (A1) for substitution
 $r^3 = 735$
 $r = 9.024623926$ (A1) for correct value
 $r = 9.02$
 $r = 9.02 \times 10^1 \text{ cm}$ A1 N4 [4]
4. (a) The volume
 $= \frac{1}{3}\pi r^2 h$ (M1) for valid approach
 $= \frac{1}{3}\pi(27)^2(27)$
 $= 20611.9894$ (A1) for correct value
 $= 20600$
 $= 2.06 \times 10^4 \text{ cm}^3$ A1 N3 [3]
- (b) $V = 27\left(\frac{4}{3}\pi r^3\right)$ (M1) for setting equation
 $4(20611.9894) = 36\pi r^3$ (A1) for substitution
 $r^3 = 729$
 $r = 9$ A1
The ratio
 $= 27:9$
 $= 3:1$ A1 N4 [4]

Exercise 51

1. (a) The total surface area
 $= 4\pi r^2$
 $= 4\pi(15)^2$ (A1) for substitution
 $= 2827.433388$
 $= 2830 \text{ cm}^2$ A1 N2 [2]
- (b) $V = 4\pi r^2$
 $2827.433388 \times (1 + 30\%) = 4\pi r^2$ (M2) for setting equation
 $r^2 = 292.5$ (M1) for valid approach
 $r = 17.10263138$
 $r = 17.1 \text{ cm}$ A1 N4 [4]
2. (a) The total surface area
 $= 4\pi r^2$
 $= 4\pi(14)^2$ (A1) for substitution
 $= 2463.00864$
 $= 2460 \text{ cm}^2$ A1 N2 [2]
- (b) $V = 4\pi r^2$
 $2463.00864 \times (1 + 15\%) = 4\pi r^2$ (M2) for setting equation
 $r^2 = 225.4$ (M1) for valid approach
 $r = 15.01332741$
 The percentage increase

$$= \frac{\frac{4}{3}\pi(15.01332741)^3 - \frac{4}{3}\pi(14)^3}{\frac{4}{3}\pi(14)^3} \times 100\%$$
 M1
 $= 23.32376089\%$
 $= 23.3\%$ A1 N5 [5]

3. (a) The total surface area
 $= 2\pi r^2 + 2\pi rh$
 $= 2\pi(18)^2 + 2\pi(18)(8)$ (A1) for substitution
 $= 2940.530724$
 $= 2940 \text{ cm}^2$ A1 N2 [2]
- (b) Increase in total surface area
 $= 2(2rh)$ (M1) for valid approach
 $= 2(2)(18)(8)$
 $= 576$ (A1) for correct value
 The percentage increase
 $= \frac{576}{2940.530724} \times 100\%$ M1
 $= 19.58830069\%$
 $= 19.6\%$ A1 N4 [4]
4. (a) The total surface area
 $= \pi r^2 + \pi rl$
 $= \pi(7)^2 + \pi(7)(25)$ (A1) for substitution
 $= 703.7167544$
 $= 704 \text{ cm}^2$ A1 N2 [2]
- (b) The vertical height h
 $= \sqrt{25^2 - 7^2}$ (M1) for valid approach
 $= 24$
 Increase in total surface area
 $= 2\left(\frac{1}{2}(2r)(h)\right)$ (M1) for valid approach
 $= 2rh$
 $= 2(7)(24)$
 $= 336$ (A1) for correct value
 The percentage increase
 $= \frac{336}{703.7167544} \times 100\%$ M1
 $= 47.74648293\%$
 $= 47.7\%$ A1 N5 [5]

Exercise 52

1. (a) The volume
- $$= \frac{2}{3} \pi r^3 \quad \text{(M1) for valid approach}$$
- $$= \frac{2}{3} \pi (10)^3 \quad \text{(A1) for substitution}$$
- $$= 2094.395102$$
- $$= 2090 \text{ cm}^3 \quad \text{A1 N3} \quad [3]$$
- (b) The total surface area
- $$= 2\pi r^2 + \pi r^2 \quad \text{(M1) for valid approach}$$
- $$= 3\pi r^2$$
- $$= 3\pi (10)^2 \quad \text{(A1) for substitution}$$
- $$= 942.4777961$$
- $$= 942 \text{ cm}^2 \quad \text{A1 N3} \quad [3]$$
- (c) $V = \frac{1}{3} \pi r^2 h$ (M1) for setting equation
- $$4(2094.395102) = \frac{1}{3} \pi \left(\frac{38}{2} \right)^2 \quad \text{(A1) for substitution (OV)}$$
- $$\text{OV} = 22.16066482$$
- $$\text{OV} = 22.2 \text{ cm} \quad \text{A1 N3} \quad [3]$$
- (d) The slant height l
- $$= \sqrt{22.16066482^2 + 19^2} \quad \text{(M1) for valid approach}$$
- $$= 29.19066743$$
- $$\cos \hat{A}\hat{V}\hat{B} = \frac{VA^2 + VB^2 - AB^2}{2(VA)(VB)} \quad \text{(M1) for cosine rule}$$
- $$\cos \hat{A}\hat{V}\hat{B} = \frac{29.19066743^2 + 29.19066743^2 - 38^2}{2(29.19066743)(29.19066743)} \quad \text{(A1) for substitution}$$
- $$\hat{A}\hat{V}\hat{B} = 81.21792258$$
- $$\hat{A}\hat{V}\hat{B} = 81.2^\circ \quad \text{A1 N4} \quad [4]$$

(e) The total surface area

$$= \pi r^2 + \pi r l$$

(M1) for valid approach

$$= \pi(19)^2 + \pi(19)(29.19066743)$$

(A1) for substitution

$$= 2876.513489$$

A1 N3

$$= 2880 \text{ cm}^2$$

[3]

2. (a) The volume
 $= A_1 h_1$ (M1) for valid approach
 $= (260)(100)$
 $= 26000 \text{ cm}^3$ A1 N2 [2]
- (b) The total surface area
 $= 2A_1 + ph_1$ (M1) for valid approach
 $= 2(260) + (62)(100)$ (A1) for substitution
 $= 6720 \text{ cm}^2$ A1 N3 [3]
- (c) $V = \frac{1}{3} A_2 h_2$ (M1) for setting equation
 $26000 = \frac{1}{3} (AD)^2 (40)$ (A1) for substitution
 $AD = 44.15880433$
 $AD = 44.2 \text{ cm}$ A1 N3 [3]
- (d) $\tan \hat{VMO} = \frac{OV}{OM}$ (M1) for tangent ratio
 $\tan \hat{VMO} = \frac{OV}{\frac{1}{2} AD}$
 $\tan \hat{VMO} = \frac{40}{\frac{1}{2} (44.15880433)}$ (A1) for substitution
 $\hat{VMO} = 61.10195875$
 $\hat{VMO} = 61.1^\circ$ A1 N3 [3]

(e)	OA $= \sqrt{OM^2 + AM^2}$ $= \sqrt{OM^2 + OM^2}$ $= \sqrt{2 \left(\frac{1}{2} (44.15880433) \right)^2}$ $= 31.22498999$ $\tan \hat{V}AO = \frac{OV}{OA}$ $\tan \hat{V}AO = \frac{40}{31.22498999}$ $\hat{V}AO = 52.02352051$ $\hat{V}AO = 52.0^\circ$	<p>(M1) for valid approach</p> <p>(A1) for correct value</p> <p>(M1) for tangent ratio</p> <p>(A1) for substitution</p> <p>A1 N5</p>
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[5]

3. (a) $\sin \hat{O\hat{V}A} = \frac{OA}{VA}$ (M1) for sine ratio
 $\sin \hat{O\hat{V}A} = \frac{40}{104}$
 $\hat{O\hat{V}A} = 22.61986495$
 $\hat{O\hat{V}A} = 22.6^\circ$ A1 N2 [2]
- (b) The total surface area
 $= \pi r^2 + \pi r l$ (M1) for valid approach
 $= \pi(40)^2 + \pi(40)(104)$ (A1) for substitution
 $= 18095.57368$
 $= 18100 \text{ cm}^2$ A1 N3 [3]
- (c) The vertical height h
 $= \sqrt{104^2 - 40^2}$ (M1) for valid approach
 $= 96$
The volume
 $= \frac{1}{3} \pi r^2 h$ (M1) for valid approach
 $= \frac{1}{3} \pi(40)^2(96)$ (A1) for substitution
 $= 160849.5439$
 $= 161000 \text{ cm}^3$ A1 N4 [4]
- (d) $\tan \hat{O\hat{V}A} = \frac{R}{H}$ (M1) for tangent ratio
 $\therefore \frac{40}{96} = \frac{R}{H}$
 $R = \frac{5}{12} H$ A1 N2 [2]

(e) $V = \frac{1}{3}\pi R^2 H$ (M1) for setting equation

$$\frac{160849.5439}{2} = \frac{1}{3}\pi \left(\frac{5}{12} H\right)^2 (H)$$

(A1) for substitution

$$80424.77193 = \frac{25}{432}\pi H^3$$

$$H^3 = 442368$$

(M1) for valid approach

$$H = 76.19525049$$

$$R = \frac{5}{12}(76.19525049)$$

(M1) for substitution

$$R = 31.74802104$$

Thus, $H = 76.2$ cm and $R = 31.7$ cm. A2 N6

[6]

4. (a) $\cos \hat{O\hat{V}A} = \frac{VO}{VA}$ (M1) for cosine ratio
 $\cos \hat{O\hat{V}A} = \frac{56}{70}$
 $\hat{O\hat{V}A} = 36.86989765$
 $\hat{O\hat{V}A} = 36.9^\circ$ A1 N2 [2]
- (b) OA
 $= \sqrt{70^2 - 56^2}$ (M1) for valid approach
 $= 42$
 $AD^2 = OA^2 + OD^2$ (M1) for setting equation
 $AD^2 = OA^2 + OA^2$
 $AD^2 = 42^2 + 42^2$ (A1) for substitution
 $AD = 59.39696962$
 $AD = 59.4 \text{ cm}$ A1 N4 [4]
- (c) The volume
 $= \frac{1}{3} A_1 h_1$ (M1) for valid approach
 $= \frac{1}{3} (AD)^2 (VO)$
 $= \frac{1}{3} (59.39696962)^2 (56)$ (A1) for substitution
 $= 65856 \text{ cm}^3$ A1 N3 [3]
- (d) $V = \frac{1}{3} A_2 h_2$ (M1) for setting equation
 $\frac{65856}{2} = \frac{1}{3} (x)(56 \times 2^{-\frac{1}{3}})$ (A1) for substitution
 $x = 2222.500732$ (A1) for correct value
 y
 $= AD^2$
 $= 42^2 + 42^2$
 $= 3528$ (A1) for correct value
 $\therefore x : y$
 $= 2222.500732 : 3528$ (M1) for valid approach
 $= 1 : 0.630$ A1 N6 [6]